BBU in a blow-out regime: a proposal for an experiment

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Summary

• We propose to study systematically the BBU in a blow-out regime with high beam loading and high efficiency

• Could be a follow up to the E200 experiment in FACET-II
Transverse hosing (beam beak-up, BBU) in plasma

- This plot is for illustration only
- Courtesy of Warren Mori (UCLA)
- Observed in some runs of 3d simulations

- For reference, see https://arxiv.org/abs/1602.05260
Our BBU analysis is based on the following paper:

Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator

C. B. Schroeder,1 D. H. Whittum,2 and J. S. Wurtele1,3
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(Received 1 April 1998)

Transverse wake function of a plasma channel:

\[ W_\perp = \frac{8\Delta z}{b^4}; \quad (\Delta z << b, k_p^{-1}) \]

\( b \) – plasma channel radius
Transverse wake for an arbitrary (large) bubble

- Based on the following paper:

  PHYSICS OF PLASMAS 13, 056709 (2006)

  A nonlinear theory for multidimensional relativistic plasma wave wakefields

  W. Lu, C. Huang, M. Zhou, and M. Tzoufras
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  (Received 8 November 2005; accepted 12 April 2006; published online 26 May 2006)

  A nonlinear kinetic theory for multidimensional plasma wave wakes with phase velocities near the speed of light is presented. This theory is appropriate for describing plasma wakes excited in the so-called blowout regime by either electron beams or laser pulses where the plasma electrons move predominantly in the transverse direction. The theory assumes that all electrons within a blowout region are accelerated by the laser pulse to near relativistic energies.

- And, on the Panofsky-Wenzel theorem...

- We request help from simulation groups to confirm our findings.
We start with the Lu plasma bubble equation (LE)

We assume the driving bunch intense enough to produce an electron-free plasma bubble with radius \( R \gg k_p^{-1} \). According to Lu et al.:

\[
\frac{r_b^2}{2} + 2r_b + 1 = \frac{4\lambda(z)}{r_b};
\]

\( r_b(z) \) -- plasma bubble boundary
The bubble is almost spherical, with its radius independent of the bunch length. when $R \gg s$:

$$R \sim 1.1 q^{1/3} = 1.1 \left( \frac{N_d}{n_e} \right)^{1/3}$$

$$N_i = \frac{4}{3} R^3 n_e = 5 N_d$$
LE, wakes

Longitudinal (from the Lu equation): \( W_\parallel = \frac{4}{r_b^2}; \quad (z << r_b, k_p^{-1}) \)

(similar to a dielectric channel and periodic array of cavities)

For reference, see: A. V. Fedotov, R. L. Gluckstern, and M. Venturini (PRST-AB 064401 (1999))

Transverse:

\[
W_\perp \approx \frac{2}{r_b^2} \int W_\parallel dz = \frac{8\Delta z}{r_b^4}; \quad (\Delta z << r_b, k_p^{-1})
\]

\( r_b(z) \) -- local bubble radius at bunch location, \( z \)

(Panofsky-Wenzel, true for a dielectric channel, array of cavities and resistive wall)

For reference, see Karl Bane, SLAC-PUB-9663

Needs to be confirmed by 3d simulations!
BBU growth length (from C. Schroeder)

\[
\frac{X(z, \tau)}{X_0} \approx \frac{3^{1/4}}{2^{3/2} \pi^{1/2}} \left( \frac{\gamma_0}{\gamma} \right)^{(1-\alpha)/2} \exp\left(\frac{A_e}{A_e^{1/2}}\right) \times \cos\left[ \theta - \frac{A_e}{3^{1/2}} + \frac{\pi}{12} \right],
\]

\[\alpha = \frac{1}{2}\]

where \( \gamma_0 \) is the initial electron wave number at injection. Asymptotically, \( A_e \rightarrow (z/L_e)^{\alpha/3} \), with the instability growth length,

\[
L_e = \frac{2^{5/\alpha}}{3^{9/2\alpha}} \left( \frac{I_o}{I} \right)^{1/\alpha} \left[ \frac{\alpha g^{1-\alpha} \gamma_0^\alpha k_0 R^2}{\kappa_1 (\omega_p \tau)^2} \right]^{1/\alpha}. \tag{18}
\]

For example, if \( \alpha = 1/2 \), then the growth rate scales as \( L_e \propto (I/I_o)^{-2} (\omega_p \tau)^{-4} \), a more favorable scaling than
Growth length

\[ L_e \propto \left( \frac{N_d}{N_w} \right)^2 \left( \frac{r_b^2}{\kappa_1 \sigma_z} \right)^2 \frac{d\gamma}{dz} \]

\[ r_b(\xi) = \sqrt{1 - \xi^2}; \quad \frac{d\gamma}{dz} = E_0 \xi \]

\[ \kappa_m = \frac{\omega_p^2}{c^2} \left[ \frac{K_m(R)}{RK_{m+1}(R)} \right] \left[ 1 + \frac{RK_m(R)}{2(m + 1)K_{m+1}(R)} \right]^{-1}, \]

\[ z := 0, 0.01, 0.99 \]

\[ r(z) := \sqrt{1 - z^2} \]

\[ F(z) := \left( \frac{r(z)^2}{\kappa \ell(r(z))} \right)^2 z \]

Maximum growth length

\[ \xi \approx 0.4 \]
E200 proposed parameters (FACET-II)

Drive Beam: E = 10 GeV, I_{peak}=15 kA
\( \beta = 89.61 \text{ cm} \), \( \alpha = 0.0653 \),
\( \sigma_r = 21.17 \mu m \), \( \sigma_z = 12.77 \mu m \),
\( N = 1.0 \times 10^{10} \) (1.6 nC),
\( \varepsilon_N = 10 \mu m \)

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
\( \beta = 89.61 \text{ cm} \), \( \alpha = 0.0653 \),
\( \sigma_r = 21.17 \mu m \), \( \sigma_z = 6.38 \mu m \),
\( N = 0.3 \times 10^{10} \) (0.48 nC),
\( \varepsilon_N = 10 \mu m \)

Distance between two bunches: 150 \( \mu m \)

Plasma Density: \( 4.0 \times 10^{16} \text{ cm}^{-3} \) (with ramps)

Courtesy of Chan Joshi
Plasma and beam density with on-axis $E_z$ line out

Beam Energy

Courtesy of Chan Joshi
Witness bunch with an initial offset, $X_0$

- Let’s look for the length, $z$, needed to increase the initial betaron oscillation amplitude by a factor of 10

\[
\frac{X(z, \tau)}{X_0} \approx \frac{3^{1/4}}{2^{3/2} \pi^{1/2}} \left( \frac{\gamma_0}{\gamma} \right)^{(1-\alpha)/2} \frac{\exp(A_e)}{A_e^{1/2}} \cos \left[ \theta - \frac{A_e}{3^{1/2}} + \frac{\pi}{12} \right],
\]

\[
A_e \approx 4.6
\]

\[
z = A_e^6 L_e \approx 60 \text{ cm}
\]

For $\xi \approx 0.7$

Numbers need to be confirmed!!!
Transverse beam break-up (head-tail instability)

- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

**Short-range transverse wake**

\[ W_{\perp}(z) \sim \frac{Z_0 c z}{a^4} \]

- Transverse stability of a beam with initial offset of \( \sigma_y \)
  - no energy spread assumed in the beam
  - emittance with respect to the beam axis is shown
  - acceptable for ILC (top)
  - would be intolerable for CLIC (bottom)

\( a \approx 35 \text{ mm (ILC)} \)
\( a \approx 3.5 \text{ mm (CLIC)} \)
\( a \sim 0.1 \text{ mm (PWFA)} \)
CLIC strategy: BNS damping + μm alignment of cavities

Achieving Beam Stability

- Transverse wakes act as defocusing force on tail
  \[ \Rightarrow \text{beam jitter is exponentially amplified} \]
- BNS (Balakin, Novokhatksy, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end
Strategy was also used at the SLC...

Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo $2\pi$. The dashed curves indicate the trajectory of the bunch head.

Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].
The BNS energy spread is:

\[ \frac{\delta p}{p} \bigg|_{BNS} \geq \frac{5}{r_t^2} \left( \frac{P_w}{P_d} \right)^2 \]
Consequences of this BNS damping

• The transformer ratio, $r_t$, is typically ~2

• Thus,

\[ \frac{\delta p}{p} \bigg|_{BNS} \geq \left( \frac{P_w}{P_d} \right)^2 \]

• Example: for a 10% power transfer (beam loading) between the drive and the witness beam, one needs to have a 1% energy spread for BNS damping.

• This needs to be confirmed by computer simulations!
Experimental proposal

• Test BBU models and the required BNS energy spread by injecting the E200 witness bunch off axis and with controlled energy spread 0 – 10%

• Observe amplitude growth of betatron oscillations as a function of beam loading and bunch longitudinal position inside the bubble.