Discussion on Hosing Instability
In the Blow-Out PWFA

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Nonlinear Plasma Wake Field

Zeroth order problem: Beam Loading

Superposition is Not Satisfied for Strong Beam Load.

First order problem: Beam Hosing


* C. Huang et. al., PRL 99, 255001 (2007)
Hosing Instability in the Bubble

\[ \partial^2_{ss} x_b + k^2_{\beta} x_b = k^2_{\beta} x_c \]
\[ x''_c + c_r c_\psi \omega_0^2 x_c = c_r c_\psi \omega_0^2 x_b \]

**Nonlinear Equation!**

\[ k_{\beta} = k_p / \sqrt{2 \gamma}, \quad \omega_0 = k_p / \sqrt{2} \]
\[ c_r \equiv n_b R^2_b / r^2_0 \]
\[ c_\psi \equiv 1 / (1 + \psi_0) \]

**E-folding for the Growth Rate**

\[ 1.3 \left[ c_r c_\psi (k_{\beta} s)(\omega_0 \xi)^2 \right]^{1/3} \]

* C. Huang et. al., PRL 99, 255001 (2007)
Mitigating Hosing Instability

**BNS Damping**

Longitudinally correlated energy spread

\[
\frac{\partial^2 X_b}{\partial t^2} + \frac{\omega_{\beta}^2}{\omega_{\beta,0}} \left( \epsilon + \kappa_1 \Delta \gamma^2 \right) \frac{\partial X_b}{\partial t} + \omega_{\beta}^2 \left( 1 + \kappa_2 \Delta \gamma^2 \right) (X_b - X_c) = 0
\]

* T. Mehrling et al., PRL 118, 174801 (2017)
Mitigating Hosing Instability

What about the trailing beam?

**Drive Beam:**
- $E = 10$ GeV,
- $I_{\text{peak}} = 15$ kA,
- $\sigma_r = 3.65 \ \mu m$, $\sigma_z = 12.77 \ \mu m$,
- $N = 1.0 \times 10^{10}$ (1.6 nC), $\varepsilon_N = 50 \ \mu m$

**Trailing Beam:**
- $E = 10$ GeV,
- $I_{\text{peak}} = 9$ kA,
- $\sigma_r = 3.65 \ \mu m$, $\sigma_z = 6.38 \ \mu m$,
- $N = 4.33 \times 10^9$ (0.69 nC), $\varepsilon_N = 50 \ \mu m$ (transversely offset by 1 \mu m)

**Distance between two bunches:** 150 \mu m

**Plasma Density:** $4.0 \times 10^{16}$ cm$^{-3}$
Mitigating Hosing Instability

What about the trailing beam?

**Drive Beam:**\[ E = 10 \text{ GeV}, I_{\text{peak}} = 15 \text{ kA} \]
\[ \sigma_r = 3.65 \text{ µm}, \sigma_z = 12.77 \text{ µm}, \]
\[ N = 1.0 \times 10^{10} (1.6 \text{ nC}), \varepsilon_N = 50 \text{ µm} \]

**Trailing Beam:**\[ E = 10 \text{ GeV}, I_{\text{peak}} = 9 \text{ kA} \]
\[ \sigma_r = 3.65 \text{ µm}, \sigma_z = 6.38 \text{ µm}, \]
\[ N = 4.33 \times 10^{9} (0.69 \text{ nC}), \varepsilon_N = 50 \text{ µm} \]
(transversely offset by 1 µm)

**Distance between two bunches:** 150 µm

**Plasma Density:** \( 4.0 \times 10^{16} \text{ cm}^{-3} \)

**10% Energy Chirp**
Mitigating Hosing Instability

\[ \xi = -\sigma_z \]

\[ \xi = 0 \]

\[ \xi = \sigma_z \]

10% Energy Chirp

Overloading the Wake can compensate the chirp.
Drive Beam: $E = 10$ GeV, $I_{\text{peak}} = 15$ kA
$\sigma_r = 0.516 \, \mu m$, $\sigma_z = 12.77 \, \mu m$, 
$N = 1.0 \times 10^{10}$ (1.6 nC), $\varepsilon_N = 1 \, \mu m rad$

Trailing Beam: $E = 10$ GeV, $I_{\text{peak}} = 9$ kA
$\sigma_r = 0.516 \, \mu m$, $\sigma_z = 6.38 \, \mu m$, 
$N = 4.33 \times 10^9$ (0.69 nC), $\varepsilon_N = 1 \, \mu m rad$ (transversely offset by 1 $\mu m$)

Distance between two bunches: 150 $\mu m$
Plasma Density: $4.0 \times 10^{16}$ cm$^{-3}$ (Hydrogen)
Killing the Hosing Instability

**Blowout PWFA**

**Plasma Ion Motion** (Talk on Thursday)

Electron beam

**QEP02-XZ**

Time = 5.00 [1/\(\omega_p\)]
Killing the Hosing Instability

\[ \xi = -\sigma_z \]

\[ \xi = 0 \]

\[ \xi = \sigma_z \]

**Drive Beam**: \( E = 10 \text{ GeV}, I_{\text{peak}} = 15 \text{ kA} \)
\( \sigma_r = 0.516 \mu\text{m}, \sigma_z = 12.77 \mu\text{m} \),
\( N = 1.0 \times 10^{10} \text{ (1.6 nC)}, \quad \varepsilon_N = 1 \mu\text{mrad} \)

**Trailing Beam**: \( E = 10 \text{ GeV}, I_{\text{peak}} = 9 \text{ kA} \)
\( \sigma_r = 0.516 \mu\text{m}, \sigma_z = 6.38 \mu\text{m} \),
\( N = 4.33 \times 10^9 \text{ (0.69 nC)}, \quad \varepsilon_N = 1 \mu\text{mrad} \)
(transversely offset by 1 \mu\text{m})