Saturation of the beam-hosing instability in quasi-linear plasma-wakefield accelerators

Remi Lehe¹, Carl B. Schroeder¹, Eric Esarey¹, Jean-Luc Vay¹, Wim P. Leemans¹

¹ Lawrence Berkeley National Laboratory, USA
Outline

- Context: the beam-hosing instability
- Saturation of the instability with a linear betatron chirp
- Betatron chirp in the quasi-linear regime
For long-distance laser/plasma wakefield acceleration, the beam has **unstable betatron oscillations**

Potential issue for **preservation of emittance** in e.g. prospective plasma-based colliders.
# Equation of beam-hosing instability

## Equation of the oscillations for a flat-top bunch

\[
\partial_z^2 x_c(\xi, z) + k_B^2 x_c(\xi, z) = k_c^2 \int_0^\infty \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \kappa_p d\xi'
\]

(Acceleration effects neglected)

- \( x_c \): off-axis position of the bunch, at a given slice
- \( z \): propagation distance
- \( \xi \): head-to-tail slice coordinate along the bunch
Equation of beam-hosing instability

Equation of the oscillations for a flat-top bunch

\[ \partial_z^2 x_c(\xi, z) + k_\beta^2 x_c(\xi, z) = k_c^2 \int_0^\xi \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \kappa_p d\xi' \]

Betatron oscillations in an unperturbed bubble/wakefield

Perturbation of the bubble/wakefield by the part of the bunch that is ahead of \( \xi \)

If \( k_\beta \) is independent of \( \xi \) (constant along the bunch), the right-hand side (due to the bunch which is ahead) oscillates at the resonant frequency of the left-hand side

\[ \rightarrow \text{Growing instability} \]
Equation of beam-hosing instability

Equation of the oscillations for a flat-top bunch

\[ \partial_z^2 x_c(\xi, z) + k_\beta^2 x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(k_p(\xi' - \xi)) x_c(\xi', z) \ k_p \ d\xi' \]

The above equation is valid for:
• LWFA and PWFA
• in the blow-out and quasi-linear regime (under certain conditions)
  with different expressions for \( k_\beta, k_p, k_c \)

### Bubble regime

\[ k_p = \frac{r_{\text{adiab}}}{r_b} \frac{k_p}{\sqrt{2(1 + \psi_0)}} \]
\[ k_\beta = \frac{k_p}{\sqrt{2\gamma}} \]
\[ k_c^2 = \frac{n_b k_p^2}{n_p 2\gamma} \]

*Huang, PRL, 2007*

### Quasi-linear regime

\[ k_p = k_p \]
\[ k_c^2 = \frac{k_p^2}{2\gamma} \]
\[ k_\beta = \frac{k_p^2}{2\gamma} \left( \frac{n_d k_p L_d}{n_p} \sin(k_p(L - \xi)) + \frac{n_b}{n_p} \int_\xi^0 \sin(k_p(\xi' - \xi)) k_p d\xi' \right) \]

*driver beam loading*
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Using a betatron variation to reduce beam-hosing

\[ \partial_z^2 x_c(\xi, z) + k_\beta(\xi)^2 x_c(\xi, z) = k_c^2 \int_0^\infty \sin(\kappa_p(\xi - \xi'))x_c(\xi', z)\kappa_p d\xi' \]

Introducing a head-to-tail variation in betatron frequency should mitigate the instability. (e.g. Balakin et al., 1983)

In the blow-out regime, the betatron variation can be generated with an energy chirp. (e.g. Mehring et al., PRL, 2017)
Betatron variation: auto-phasing condition

\[ \frac{\partial^2 x_c(\xi, z)}{\partial z^2} + k_\beta(\xi)^2 x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(k_p(\xi' - \xi)) x_c(\xi', z) k_p d\xi' \]

**Auto-phasing condition**

For betatron oscillations with a constant amplitude and frequency:

\[ k_\beta(\xi)^2 - k_\beta(0)^2 = k_c^2 \int_\xi^0 \sin(k_p(\xi' - \xi)) k_p d\xi' \]

\[ k_\beta(\xi) \approx k_{\beta,0} + \frac{k_c^2 k_p^2}{2 k_{\beta,0}} \xi^2 \]

Requires quadratic variation; difficult in practice

However, this condition is somewhat **restrictive** because it searches **exclusively** for conditions in which the oscillations have a **constant** amplitude.
Betatron variation: linear chirp

\[
\frac{\partial^2}{\partial z^2} x_c(\xi, z) + k_\beta(\xi)^2 x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \kappa_p d\xi'
\]

Instead: linear chirp

\[
k_\beta(\xi) = k_{\beta,0} + (\partial_\xi k_\beta) \xi
\]

→ Several analytical solutions in different regimes relevant for conventional accelerators show mitigation as a function of \((\partial_\xi k_\beta)\)
  
  • Stupakov, SLAC report, 1997
  • Chernin & Mondeli, *Particle Accelerators*, 1989

→ Recently: analytical solution in regime relevant for plasma accelerator

*Lehe et al., submitted for publication*

Valid for small chirp

Interestingly: depends on the sign of the chirp
Positive chirp: saturation

\[ L_{\text{sat}} = \left( \frac{k_c^2 \kappa_p^2}{k_{\beta,0} |\partial_{\xi} k_{\beta}|^3 |\xi|} \right)^{1/2} \]

Standard beam-hosing (no chirp)

\[ x_c(\xi, z) = \delta x \cos \left( k_{\beta} z - \frac{3}{4} N(\xi, z) + \frac{\pi}{12} \right) e^{\frac{3\sqrt{3}}{4} N(\xi, z)} \]

\[ N(\xi, z) = \left( \frac{k_c^2 \kappa_p^2 |\xi|^2 z}{k_{\beta,0}} \right)^{1/3} \]

Asymptotic saturated solution

\[ x_c(\xi, z) = \delta x \cos \left[ k_{\beta}(\xi) z - \varphi(z) \right] e^{\sqrt{2}N_{\text{sat}}}(\xi) \]

\[ N_{\text{sat}}(\xi) = \left( \frac{k_c^2 \kappa_p^2 |\xi|}{k_{\beta,0} (\partial_{\xi} k_{\beta})} \right)^{1/2} \]
Negative chirp: saturation and slow decay

\[ L_{sat} = \left( \frac{k_c^2 \kappa_p^2}{k_{\beta,0} |\partial_\xi k_\beta|^3 |\xi|} \right)^{1/2} \]

Standard beam-hosing (no chirp)

\[ x_c(\xi, z) = \delta x \frac{\cos \left( k_\beta z - \frac{3}{4} N(\xi, z) + \frac{\pi}{12} \right)}{(6\pi)^{1/2} N(\xi, z)^{1/2}} e^{\frac{3\sqrt{3}}{4} N(\xi, z)} \]

Asymptotic solution

\[ x_c(\xi, z) = -\delta x \frac{\sin(k_{\beta,0}z)}{(32\pi^2)^{1/4} N_{sat}(\xi)^{-1/2}} e^{\frac{\sqrt{2}N_{sat}(\xi)}{|(\partial_\xi k_\beta) z \xi|}} \]

\[ + \delta x \frac{\cos[k_\beta(\xi)z - \varphi(z)]}{\left(\frac{\pi^2}{2}\right)^{1/4} N_{sat}(\xi)^{1/2}} \cos \left( \sqrt{2}N_{sat}(\xi) - \frac{\pi}{4} \right) \]
Summary

- The beam-hosing instability can severely **degrade** the emittance.

- A **positive chirp** in betatron frequency causes the instability to saturate.

- A **negative chirp** in betatron frequency causes the instability to decay.

- In both case (positive and negative), the instability is much **less severe** than predicted by **standard scaling**, which assumes constant betatron frequency.
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Generating a betatron chirp in different regimes

\[ k_\beta = \frac{K}{\sqrt{\gamma}} \quad F_{foc} = -mc^2K^2r \]

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<thead>
<tr>
<th>Blow-out regime</th>
<th>Quasi-linear regime</th>
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- Focusing force is independent of \( \xi \)
- Chirp requires a (correlated) energy spread

- Focusing force naturally depends on \( \xi \)
- No energy spread required

→ Confirm saturation in PIC simulations?
The code FBPIX (Fourier-Bessel PIC)

- Spectral quasi-cylindrical Particle-In-Cell algorithm (azimuthal mode decomposition)
- Runs on GPU and (multi-core) CPU
- Open-source: [github.com/fbpic/fbpic](https://github.com/fbpic/fbpic)
  Documentation: [fbpic.github.io](https://fbpic.github.io)

Several useful features for plasma acceleration:

- Intrinsic mitigation of Numerical Cherenkov Radiation (NCR)
- Supports the boosted-frame technique
- Calculation of initial space-charge fields
- Field ionization physics (ADK model)
The beam-hosing instability is not cylindrically symmetric.

This asymmetry can be decomposed into azimuthal modes (m=0: cylindrical symmetric, m=1: dipole mode, m=2: quadrupole mode, etc.)

If the centroid offset is small compared to the beam radius, the beam hosing instability excites predominantly the mode m=1. (i.e. modes m>1 are negligible)

FBPIC simulates modes m=0 and m=1, and thus captures the beam-hosing instability in this case (and is much faster than a full 3D Cartesian code).
Simulation results

Simulation setup

- **Witness bunch:**
  - No energy spread
  - Triangular longitudinally (flattens the Ez field)
  - matched K-V distribution

- **Driver:**
  - either laser or bunch,
  - in the linear regime

Parameters

\[
\begin{align*}
  n_p &= 2 \times 10^{17} \text{ cm}^{-3} \\
  \gamma_{bunch} &= 200 \\
  r_{bunch} &= 3 \mu m \\
  \ell_{bunch} &= 15 \mu m \\
  a_0 &= 0.4 \\
  w_0 &= 24 \mu m \\
  \tau &= 20 \text{ fs} \\
  n_d &= 0.7 n_p \\
  r_d &= 4 \mu m \\
  \ell_d &= 3 \mu m
\end{align*}
\]
Simulation results

Simulation setup

- **Witness bunch:**
  
  No energy spread
  
  Triangular longitudinally (flattens the Ez field)
  
  matched K-V distribution
  
- **Driver:**
  
  either laser or bunch,
  
  in the linear regime

Parameters

\[ n_p = 2 \times 10^{17} \text{ cm}^{-3} \quad \gamma_{\text{bunch}} = 200 \quad r_{\text{bunch}} = 3 \mu\text{m} \quad \ell_{\text{bunch}} = 15 \mu\text{m} \]

- **Laser-driven:** \[ a_0 = 0.4 \quad w_0 = 24 \mu\text{m} \quad \tau = 20 \text{ fs} \]
- **Beam-driven:** \[ n_d = 0.7 n_p \quad r_d = 4 \mu\text{m} \quad \ell_d = 3 \mu\text{m} \]
Simulation results

Simulation setup

- **Witness bunch:**
  - No energy spread
  - Triangular longitudinally (flattens the Ez field)
  - matched K-V distribution

- **Driver:**
  - either laser or bunch, in the linear regime

Beam-driven case

Laser-driven case
Conclusion

- New analytical formula, shows that the beam-hosing instability saturates in the presence of a betatron frequency chirp along the witness beam.

- In the quasi-linear regime, betatron chirp occurs naturally, even for a monoenergetic bunch.

- Thus, the beam-hosing instability in the quasi-linear regime is much less severe than predicted by standard hosing scalings.
Thank you for your attention

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