Power efficiency vs instability
(or, emittance vs beam loading)

Sergei Nagaitsev, Valeri Lebedev, and Alexey Burov
Fermilab/UChicago
Oct 18, 2017
We would like to thank our UCLA colleagues for discussions and computer simulations and G. Stupakov for fruitful discussions.
Our motivation for this study came from:

**A Beam Driven Plasma-Wakefield Linear Collider: PWFA-LC**

**From Higgs Factory to Multi-TeV**

J.P Delahaye / SLAC

On behalf of the E200 Collaboration


C. Joshi, W. An, C.E. Clayton, K.A. Marsh, W. Mori, N. Vafaie-Najafabadi (UCLA, Los Angeles, USA),

W. Lu (Tsinghua Univ. of Beijing, China and UCLA)

P. Muggli (MPI, Munich, Germany)

Thanks for slides from M. Hogan, E. Adli, S. Gessner
Gradient and efficiency in Linear Colliders

- High gradient acceleration requires high peak power and structures that can sustain high fields
  - Beams and lasers can be generated with high peak power
  - Dielectrics and plasmas can withstand high fields

<table>
<thead>
<tr>
<th>Acc. structures</th>
<th>Accelerating field</th>
<th>Acceleration efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Limit (MV/m)</td>
<td>By</td>
</tr>
<tr>
<td>Super-Conducting</td>
<td>ILC</td>
<td>30-40</td>
</tr>
<tr>
<td>Normal Conducting</td>
<td>CLIC Two beam</td>
<td>100</td>
</tr>
<tr>
<td>Dielectric</td>
<td>Laser driven</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>Beam driven</td>
<td></td>
</tr>
<tr>
<td>Plasma</td>
<td>Laser driven</td>
<td>10000</td>
</tr>
<tr>
<td></td>
<td>Beam driven</td>
<td>Drive beam</td>
</tr>
</tbody>
</table>

Beam-driven Plasma Wake-Field Accelerator (PWFA)

J.P. Delahaye @ MIT April 11, 2013
Plasma Acceleration (Beam-driven or Laser-driven)

- Defocusing
- Focusing ($E_1$)
- Accelerating
- Decelerating ($E_2$)
- Laser pulse or electron beam

- Two-beam, co-linear, plasma-based accelerator
- Plasma wave/wake excited by relativistic particle bunch
- Deceleration, acceleration, focusing by plasma
- Accelerating field/gradient scales as $n_e^{1/2}$
- Typical: $n_e \approx 10^{17}$ cm$^{-3}$, $\lambda_e \approx 100$ µm, $E > 10$ GV/m
- High-gradient, high-efficiency energy transformer

**Peak Field For A Gaussian Bunch:**

$$E = \frac{6GV/m}{2 \times 10^{10}} \frac{N}{\sigma_r \sigma_z}$$

**Extremely strong focusing:**

$$B_{dr} = 2 \pi e \eta_p$$

**Excellent power transfer efficiency:**

- $\eta_{\text{drive to plasma}} \sim 76\%$
- $\eta_{\text{plasma to main}} \sim 66\%$

$\eta_{\text{drive to main}} > 50\%$

$> 10$ GV/m

$> \text{MT/m}$
Is drive-to-beam 50% efficiency possible???

Conclusions

PWFA a very promising technology:

- Very high accelerating fields: effective 1 GV/m
- Excellent power efficiency (Wall-plug to beam 20%)

Great flexibility of time interval

- CW or pulsed mode of operation
- An alternative for ILC energy upgrade?

Many challenges still to be addressed:

- Beam quality preservation, efficiency, positrons?
- Ambitious test facilities: FACET and FACET2
- Feasibility addressed early next decade?

Thanks to excellent and expert collaboration: E200
Why is power efficiency important? Because power = cost
Acceleration in ILC cavities

- The ILC cavity: ~1 m long, 30 MeV energy gain; \( f_0 = 1.3 \text{ GHz} \), wave length \( \approx 23 \text{ cm} \)
- The ILC beam: 3.2 nC \( (2 \times 10^{10}) \), 0.3 mm long (rms); bunches are spaced \( \sim 300 \text{ ns (90 m)} \) apart
- Each bunch lowers the cavity gradient by \( \sim 15 \text{ kV/m} \) (beam loading 0.05%); this voltage is restored by an external rf power source (Klystron) between bunches; (\( \sim 0.5\% \text{ CLIC} \))
- Such operation of a conventional cavity is only possible because the Q-factor is \( \gg 1 \); the RF energy is mostly transferred to the beam NOT to cavity walls.
Acceleration in a blow-out regime

- The Q-factor is very low (~1) – must accelerate the trailing bunch within the same bubble as the driver!

- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.

To achieve L \(\sim 10^{34}\), bunches should have \(\sim 10^{10}\) particles (similar to ILC and CLIC). In principle, we can envision a scheme with fewer particles/bunch and a higher rep rate, but the beam loading still needs to be high for efficiency reasons.
Transverse beam break-up (head-tail instability)

- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

\[ W_\perp(z) = \frac{8z}{a^4} \]

- Transverse stability of a beam with initial offset of \( \sigma_y \)
  - no energy spread assumed in the beam
  - emittance with respect to the beam axis is shown
  \( \Rightarrow \) acceptable for ILC (top)
  \( \Rightarrow \) would be intolerable for CLIC (bottom)

\( a \approx 35 \) mm (ILC)
\( a \approx 3.5 \) mm (CLIC)
\( a \sim 0.1 \) mm (PWFA)
Case I: ~50% power efficiency
Hosing Study for FACET II: Case I

**Drive Beam:**
- $E = 10 \text{ GeV}$, $I_{\text{peak}} = 15 \text{ kA}$
- $\sigma_r = 3.65 \mu\text{m}$, $\sigma_z = 12.77 \mu\text{m}$
- $N = 1.0 \times 10^{10}$ (1.6 nC), $\varepsilon_N = 10 \mu\text{m}$

**Trailing Beam:**
- $E = 10 \text{ GeV}$, $I_{\text{peak}} = 9 \text{ kA}$
- $\sigma_r = 3.65 \mu\text{m}$, $\sigma_z = 6.38 \mu\text{m}$
- $N = 4.33 \times 10^9$ (0.69 nC), $\varepsilon_N = 10 \mu\text{m}$
  (transversely offset by 1 $\mu\text{m}$)

**Distance between two bunches:** 150 $\mu\text{m}$

**Plasma Density:** $4.0 \times 10^{16}$ cm$^{-3}$

Trailing beam centroid vs $s$ in different slices
Case II: ~25% power efficiency
Hosing Study for FACET II: Case II

Drive Beam: $E = 10$ GeV, $I_{\text{peak}} = 15$ kA
$\sigma_r = 3.65 \, \mu\text{m}$, $\sigma_z = 12.77 \, \mu\text{m}$,
$N = 1.0 \times 10^{10}$ (1.6 nC), $\varepsilon_N = 10 \, \mu\text{m}$

Trailing Beam: $E = 10$ GeV, $I_{\text{peak}} = 9$ kA
$\sigma_r = 3.65 \, \mu\text{m}$, $\sigma_z = 6.38 \, \mu\text{m}$,
$N = 4.33 \times 10^9$ (0.69 nC), $\varepsilon_N = 10 \, \mu\text{m}$
(transversely offset by 1 \( \mu\text{m} \))

Distance between two bunches: 108 \( \mu\text{m} \)
Plasma Density: $4.0 \times 10^{16} \, \text{cm}^{-3}$

Trailing beam centroid vs s in different slices
Beam breakup in various collider concepts

- **ILC**
  - Not important; bunch rf phase is selected to compensate for long wake and to minimize the momentum spread

- **CLIC**
  - Important; bunch rf phase is selected to introduce an energy chirp along the bunch for BNS damping (≈0.5% rms). May need to be de-chirped after acceleration to meet final-focus energy acceptance requirements

- **PWFA** – *the subject of our study*
  - Critical; BNS damping requires a large energy chirp (see below). De-chirping and beam transport is very challenging because of plasma stages (small beta-function in plasma ≈1 cm). In essence, requires a “final-focus” optics between every stage.
CLIC strategy: BNS damping + $< \mu m$ alignment of cavities

**Achieving Beam Stability**

- Transverse wakes act as defocusing force on tail
  $\Rightarrow$ beam jitter is exponentially amplified
- BNS (Balakin, Novokhatsky, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end

![Diagram showing the effect of BNS damping on beam stability](image)
Strategy was also used at the SLC…

Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo $2\pi$. The dashed curves indicate the trajectory of the bunch head.

Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].
We start with the Lu plasma bubble equation

- We assume the driving bunch intense enough to produce an electron-free plasma bubble with radius $R_b \gg k_p^{-1}$. According to Lu et al.:

$$r_b \frac{d^2 r_b}{d \xi^2} + 2 \left( \frac{dr_b}{d \xi} \right)^2 + 1 = \frac{2}{\pi n_0 r_b^2} \frac{dN_d}{d \xi}$$

$$E_\parallel = -2\pi n_0 e r_b \frac{dr_b}{d \xi}$$

$$R_b = \frac{L_d}{4\sqrt{2}} \sqrt[4]{\frac{8N_d}{\pi n_0 L_d^3} \left( \sqrt{\frac{8N_d}{\pi n_0 L_d^3}} + 1 - 1 \right)}$$

$$R_b \approx \left( \frac{2^7 N_d^3}{\pi^3 L_d n_0^3} \right)^{1/8}, \quad \frac{N_d}{n_0 L_d^3} \gg 1$$

Example: $N_d = 10^{10}$; $n_0 = 4 \times 10^{16}$ cm$^{-3}$; $L = 25$ μm

$$R_b k_p \approx 3.2$$
Power transfer from drive to trailing bunches

• Following M. Tzoufras et al., PRL 101, 145002 (2008)

Trapezoidal line density distribution $\rightarrow$ constant electric field

\[
P = eN_d E_d c = \frac{\pi^2}{4} e^2 n_0^2 c R_b^4
\]

\[
P_t = ecN_t E_t = \frac{\pi^2 e^2 n_0^2 c}{4} (r_{t2}^2 - r_{t1}^2) \left( \frac{R_b^4}{r_{t2}^2} + r_{t1}^2 \right)
\]

\[
\eta_P = \frac{P_t}{P} = \frac{r_{t2}^2 - r_{t1}^2}{R_b^2} \left( \frac{R_b^2}{r_{t2}^2} + \frac{r_{t1}^2}{R_b^2} \right)
\]
The power transfer efficiency of 50% and the transformer ratio of 2. For $n_0=10^{17}$ cm$^{-3}$ the drive bunch parameters are chosen to be $R_b k_p=5, L_d k_p=2.5$ yielding the decelerating field of $E_d = 50$ GV/m and $N_d=3.55 \cdot 10^{10}$. The trailing bunch parameters are: $r_{t2}=0.518 R_b, r_{t1}=0.373 R_b, E_t = 100$ GV/m, $N_t=8.86 \cdot 10^9$. 
Instability of the trailing bunch

- The Beam Break-up (BBU) instability is characterized by the ratio of the wake deflection force to the focusing force.

\[ F_r = -2\pi n_0 e^2 r \quad \text{Focusing force} \]

\[ F_t \equiv F(\xi_1) = e^2 r \int_{\xi_1-L_t}^{\xi_1} \frac{dN_t}{d\xi} W_\perp(\xi_1, \xi) d\xi \quad \text{Defocusing force (varies along bunch)} \]

- Need to find \( W_\perp(\tilde{\xi}) \) for the bubble regime.
- First, in a quasilinear regime,

\[ W_\perp = 2 \frac{k_p}{\sigma_\perp} \left( \frac{\Delta n}{n} \right) e \sin(k_p(s-s'))\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right), \quad k_p = \frac{\omega_p}{c} \]

- where \( \sigma_\perp \) is the rms size of plasma channel
- For a hollow channel \( \frac{\Delta n}{n} \sim 1 \)

\[ W_\perp \approx 2k_p^3 \sin(k_p(s-s'))\ln(2), \quad \sigma_\perp \approx k_p^{-1} \]
Wakes in the bubble regime

Longitudinal (from the Lu equation):

\[ W_\parallel = \frac{4}{r_b^2}; \quad (\Delta z << r_b, k_p^{-1}) \]

(similar to a dielectric channel and periodic array of cavities)

For reference, see: A. V. Fedotov, R. L. Gluckstern, and M. Venturini (PRST-AB 064401 (1999))

Transverse:

\[ W_\perp \approx \frac{2}{r_b^2} \int W_\parallel dz = \frac{8\Delta z}{r_b^4}; \quad (\Delta z << r_b, k_p^{-1}) \]

\( r_b(z) >> k_p^{-1} \) -- local bubble radius at bunch location, \( z \)

(This is true for a dielectric channel, array of cavities and resistive wall)

For reference, see also: Karl Bane, SLAC-PUB-9663 and S. S. Baturin and A. D. Kanareykin, PRL 113, 214801 (2014) .

Recent findings: \( \tilde{r}_b(z) \rightarrow r_b(z) + k_p^{-1} \) to account for bubble wall thickness
Our estimate for the transverse wake

\[ W_{\perp}(\xi, \xi_2) \approx \frac{8\xi}{r_b(\xi)r_b^3(\xi_2)} \theta(\tilde{\xi}), \quad \tilde{\xi} = \xi - \xi_2 \]

\[ r_b(\xi) \gg k_p^{-1} \]

- \( \theta(x) \) is the Heaviside step function.

• We believe this estimate is on the “low” side. The actual wake is likely to be greater.

• Now, let’s find the ratio of the defocusing (wake) force to the focusing force:

\[ \eta_t = -\frac{F_t}{F_r} = \frac{r_{t2}}{r_{t1}} \int_0^{L_t} d\xi \frac{L_t - \xi}{r_b^3(\xi)} \times \left[ r_{t2} \left( \frac{R_b^4}{r_{t2}^4} - 1 \right) - 2 \left( \xi \sqrt{2 \left( \frac{R_b^4}{r_{t2}^4} - 1 \right)} - r_{t2} \right) \right] \]

• Recall that

\[ \eta_P = \frac{P_t}{P} = \frac{r_{t2}^2 - r_{t1}^2}{R_b^2} \left( \frac{R_b^2}{r_{t2}^2} + \frac{r_{t1}^2}{R_b^2} \right) \]
The efficiency-instability relation

\[ \eta_t \approx \frac{\eta_P^2}{4(1-\eta_P)}, \quad \frac{r_{t2}}{R_b} \leq 0.7 \]

- This formula does not include any details of beams and plasma, being amazingly universal!
- Note: this formula is an estimate from a “low side”. On a “high side”, we estimate it as: \( \eta_t \approx \eta_P^2 / \left(4(1-\eta_P)^2\right) \)
- Example: \( \eta_P = 50\% \Rightarrow 0.125 < \eta_t < 0.25 \)
  \[ \eta_P = 25\% \Rightarrow 0.021 < \eta_t < 0.028 \]
Instability development

\[ \frac{d^2 X}{d\mu^2} + \frac{X}{1+\Delta p/p} = \frac{2\eta_t}{(1+\Delta p/p)L_t^2} \int_0^\xi X(\xi')(\xi - \xi')d\xi'. \]

\[ X = \frac{x}{\sqrt{\beta}} \sqrt{\frac{p}{p_0}}; \quad \beta = k_p^{-1} \sqrt{2\gamma} \quad d\mu = dz / \beta \]

- For \( \eta_t \ll 1 \) and \( \Delta p / p = 0 \) it was solved in:

- Approximate solutions (it’s a very good fit, <10% deviation):
  \[ \frac{A}{A_0} = \exp\left[ \frac{(\mu\eta_t)^2}{60 + 1.4(\mu\eta_t)^{1.57}} \right] \quad \mu\eta_t \ll 100 \]
  \[ \eta_t \ll 0.1 \]
  \[ \frac{\sqrt{A^2}}{A_0} = \exp\left[ \frac{(\mu\eta_t)^2}{60 + 2.2(\mu\eta_t)^{1.57}} \right] \quad \mu\eta_t \ll 100 \]
  \[ \eta_t \ll 0.1 \]
• Note that $A$ is a normalized particle amplitude. For a constant plasma density and without instability $A$ would stay constant, while the initial physical amplitude $x$ should decrease as $1 / \gamma^4$. 
Examples (FACET-II)

Plasma: $n_0 = 4 \times 10^{16}$ cm$^{-3}$, 60 cm long channel

- $p_f=10$ GeV/c for both the drive and the trailing bunches, and the final momentum of trailing bunch $p_f=21$ GeV/c, $N_d=1 \times 10^{10}$ and $N_t=4.3 \times 10^9$

  $$\eta_P = 50\%, \; \eta_t \approx 0.12, \; \mu \eta_t \approx 11.5 \; \rightarrow \; \frac{A}{A_0} \approx 5.8$$

- If one reduces the power efficiency:

  $$\eta_P = 25\%, \; \eta_t \approx 0.021, \; \mu \eta_t \approx 2 \; \rightarrow \; \frac{A}{A_0} \approx 1.3$$

- Of course, the final momentum is now $p_f=15.5$ GeV/c (for the same number of particles)

$$\delta \varepsilon_n = \frac{\delta x^2}{2 \beta_i} \gamma_i \left( \frac{A^2}{A_0^2} \right), \quad \beta_i = \frac{\sqrt{2 \gamma_i}}{k_p}$$
BNS damping

- Assume a constant long. density trailing bunch. Chromatic detuning of tail particles allows to keep amplitudes constant

\[
\frac{1}{1 + \frac{\Delta p}{p}} - \frac{2 \eta_t}{\left(1 + \frac{\Delta p}{p}\right)} L_t^2 \int_0^{\xi} \left(\xi - \xi'\right) d\xi' = 1
\]

\[
\frac{Dp(\xi)}{p} = - \eta_t \frac{\xi^2}{L_t^2}
\]

- We believe that the collider final focus optics and transitions between stages can not tolerate \(\frac{\Delta p}{p} > 1\%\), so \(\eta_t \leq 0.01\)

- This limits the power transfer efficiency to < 18%
Conclusions

• We have found a universal efficiency-instability relation for plasma acceleration. Should allow for tolerance and instability analysis without detailed computer simulations.

• We considered only the ideal “trapezoidal” distributions. Real-life distributions are worse (from the efficiency perspective).

• In a blowout regime, plasma focusing is just strong enough to keep the instability in check for low power efficiencies (<25%)
  – Even for such efficiencies, external focusing and hollow channels are not viable concepts because of transverse instability.
  – Presents obvious difficulties for positrons

• BNS damping is possible but external optical systems limit the momentum spread to ~1% max. Thus, the power efficiency (drive to trailing) can not exceed ~18%.
Summary

• We wish FACET-II success and would like to be part of its science program.
• Our conclusions require confirmation by computer simulations and by experiments, especially in regimes not covered by the Lu equations (small bubble size).