Overview of Non-Perturbative Methods for Strong-Field Physics

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Workshop: *Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime*

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[DOE Division of High Energy Physics]

Probing Nonlinear QED: Why are we here?

precision study of QFT in extreme (intensity) conditions

complementary to traditional collider physics

applications to future colliders

- classical nonlinear parameter: $\xi_0 = \frac{e\,\mathcal{E}}{m\,c\,\omega} \sim \frac{eA}{mc}$
- quantum non-linearity parameter: $\chi_0 = \frac{e\,\hbar}{m^3\,c^4}\sqrt{-\left(F_{\mu\nu}\,p^\nu\right)^2}$

new nonlinear QED regime: $\chi_0 \gg 1$

novel perturbative & non-perturbative effects: nonlinear Compton; Breit-Wheeler pair production; vacuum instability; cascades; mag. moment interaction energy; magnetic catalysis ...

- how to probe this regime with a lepton collider?
- what are the important theoretical issues to be resolved?
- lessons for/from: heavy ions; lasers; colliders; plasma; astrophysics; BSM; nonequilibrium; lattice; simulations; ...

Euler-Heisenberg (1935): QED Effective Action

non-perturbative QED effective action in a constant field:

$$\Gamma[F] = -\frac{1}{8\pi^2}\int_0^\infty \frac{d\eta}{\eta^3}\, e^{-\eta\, e\mathcal{E}_{\rm cr}} \left\{ \frac{e^2 a\, b\, \eta^2}{\tanh(eb\eta)\tan(ea\eta)} - 1 - \frac{e^2\eta^2}{3}(b^2-a^2) \right\}$$

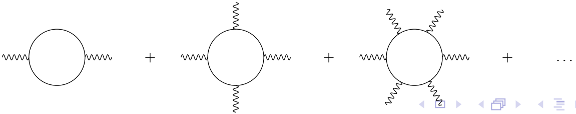
critical field: $\mathcal{E}_{\rm cr} = \frac{m^2 c^3}{e\,\hbar} \approx 10^{16}\mathrm{V/cm}$ (work done in λ_C)

$$a^2 - b^2 = \vec{\mathcal{E}}^2 - \vec{\mathcal{B}}^2 = -\frac{1}{2} F_{\mu\nu}F^{\mu\nu} \equiv -2\mathcal{F}$$

$$a\, b = \vec{\mathcal{E}}\cdot\vec{\mathcal{B}} = -\frac{1}{4} F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv -\mathcal{G}$$

$$a = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}-\mathcal{F}}\ , \quad b = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}+\mathcal{F}}$$

perturbative expansion:



Euler-Heisenberg (1935): QED Effective Action

polarization of the quantum vacuum by the external field

$$\Gamma = \frac{e^4}{360\pi^2 m^4} \int d^4x \left[(\vec{\mathcal{E}}^2 - \vec{\mathcal{B}}^2)^2 + 7(\vec{\mathcal{E}} \cdot \vec{\mathcal{B}})^2 \right] + \dots$$

- non-linear: light-light scattering; vacuum birefringence/dichroism; pair production; ...
- electric field: non-perturbative imaginary part, $e^\pm$ pair production: $\operatorname{Im}\Gamma = \mathcal{E}^2 \mathrm{Li}_2\left(\frac{\pi\, \mathcal{E}_{\rm cr}}{\mathcal{E}}\right) \approx \mathcal{E}^2 e^{-\pi \frac{\mathcal{E}_{\rm cr}}{\mathcal{E}}}$
- charge renormalization (Heisenberg)
- divergent (double) series (cf. Dyson, 1950): Borel integral representation of "multiple-gamma function"
- Weisskopf (1936): strong-field limit: $\Gamma \sim \beta_0\, F^2\, \ln(eF/m^2)$

Euler-Heisenberg (1935): QED Effective Action

- Euler's thesis (1936): "effective field theory", double-sum over fields & derivatives; Lorentz and gauge invariant local terms

$$\Gamma[A] = \frac{\hbar c}{e^2}\frac{1}{\mathcal{E}_{\rm cr}^2}\int d^4x \left[FFFF + \left(\frac{\hbar}{m\,c}\frac{\partial F}{\partial x}\right)^2 FF + \dots\right]$$

- contrast Euler/Heisenberg with Serber/Uehling (1935) linear, but space-time dependent, vacuum polarization effects
- double-sum is doubly divergent
- how to go beyond the constant field limit?

Feynman (1949) & Schwinger (1951): QED Effective Action

Feynman:

- relativistic path integral as sum over Minkowski trajectories parametrized by proper-time
- conservation of particles in proper-time $\equiv$ charge conjugation symmetry
- scattering picture of electron/positron propagation

Schwinger:

- functional/Fredholm determinants (Jost/Pais, Salam/Matthews, Salam/Strathdee, ...)
- path integral representation of log det formulation
- Dirac equation solvable for constant background field (Fock, Nambu) and for plane-wave field (Volkov)

- "non-perturbative" = semiclassical approximation

Feynman (1949-1951): QED Effective Action

Fock, Stückelberg, Nambu expressed Klein-Gordon and Dirac equation in proper-time form:

$$i\frac{\partial}{\partial u}\Phi = \frac{1}{2}\left(i\frac{\partial}{\partial x_\mu} - A_\mu\right)^2 \Phi \equiv \mathcal{H}\,\Phi$$

Feynman: amplitude for evolution in u, $\langle x|e^{-i\,\mathcal{H}\,u}|y\rangle$
One-loop effective action:

$$\Gamma[A] = -\int_0^\infty \frac{du}{u} e^{-i\,m^2 u} \int d^4x \int_{x(u)=x(0)=x} \mathcal{D}x\, e^{i\,S[x]}$$

$S[x]$: classical action for charged scalar in proper time u

$$S[x] = \int_0^u d\tau \left(\frac{1}{2}\left(\frac{dx^\mu}{d\tau}\right)^2 + \frac{e}{c}\,A_\mu \frac{dx^\mu}{d\tau}\right)$$

Klein-Gordon/Dirac equations, and also classical trajectories, solvable for constant field and plane-wave field

Schwinger (1951-1954): QED Effective Action

One-loop effective action: $\langle 0\,\sigma_1 | 0\,\sigma_2\rangle \equiv e^{i\,\Gamma[A]}$

$$\Gamma[A] = -i\ln\det\left(1 + e\,\gamma\,A\,G_+^{(0)}\right) = -\frac{i}{2}\ln\det\left(1 - e^2\,G_+^{(0)}\,\gamma\,A\,G_+^{(0)}\,\gamma\,A\right)$$

local effective Lagrangian $\mathcal{L}(x)$:

$$\begin{aligned}
\mathcal{L}(x) &= \frac{1}{2}\,i\int_0^\infty \frac{ds}{s}\,\exp(-im^2 s)\,\mathrm{tr}(x|U(s)|x) \\
U(s) &= \exp(-i\,\mathcal{H}\,s) \qquad , \qquad \mathcal{H} = \Pi_\mu^2 - \frac{1}{2}e\sigma_{\mu\nu}F_{\mu\nu}
\end{aligned}$$

local current: $\langle j^\mu(x)\rangle = \frac{\delta\Gamma}{\delta A_\mu(x)} = i\,e\,\mathrm{tr}\left(\gamma^\mu\,G_+(x,x)\right)$

scattering operator: $\mathcal{T} \equiv e\,\gamma\,A\left(1 - G_+^{(0)}\,e\,\gamma\,A\right)^{-1}$

$$\left|e^{i\,\Gamma[A]}\right|^2 = \det\left(1 + S^{(-)}\,\mathcal{T}\,S^{(+)}\,\bar{\mathcal{T}}\right)^{-1}$$

Feynman (1949) and Schwinger (1951): QED Effective Action

Volkov (1935): exact solution for plane-wave:

$$A_\mu = A\, e_\mu f(\phi) \quad , \quad \phi \equiv k \cdot x$$

effective action vanishes (Schwinger, 1951):

$$\Gamma_{\rm PW} = 0$$

effective action also vanishes for constant-crossed-field (CCF):
$\mathcal{E} \perp \mathcal{B}, |\mathcal{E}| = |\mathcal{B}|$ (Euler-Heisenberg, 1935):

$$\Gamma_{\rm CCF} = 0$$

Brézin/Itzykson (1970); Popov/Marinov (1971): QED Effective Action in Monochromatic Electric Field

monochromatic field: $\mathcal{E}(t) = \mathcal{E}\,\cos(\omega\,t)$

effective action: $\Gamma(\mathcal{E}) \to \Gamma(\mathcal{E}, \xi_0)$

classical nonlinear parameter [(adiabaticity parameter)$^{-1}$]:

$$\xi_0 = \frac{e\,\mathcal{E}}{m\,c\,\omega} \sim \frac{eA}{mc}$$

semiclassical imaginary part:

$$\mathrm{Im}\,\Gamma \approx e^{-\frac{\mathcal{E}_{\rm cr}}{\mathcal{E}} g(\xi_0)} \sim \begin{cases} e^{-\frac{\pi \mathcal{E}_{\rm cr}}{\mathcal{E}}} \quad , & \xi_0 \gg 1 \quad \text{(non-perturbative)} \\ e^{\frac{\mathcal{E}_{\rm cr}}{\mathcal{E}}\,4\,\xi_0\,\ln\xi_0} = \xi_0^{\frac{4mc^2}{\hbar\,\omega}} \quad , & \xi_0 \ll 1 \quad \text{(perturbative)} \end{cases}$$

tunneling *vs* multi-photon pair production (cf. Keldysh 1964)

Narozhny/Nikishov (1970): localized fields

sech^2 localized fields: Dirac/Klein-Gordon, and classical trajectories, solvable (hypergeometric)

$$\mathcal{E}(t) = \mathcal{E}\,\text{sech}^2(t/T) \qquad , \quad \xi_0 = \frac{e\,\mathcal{E}\,T}{m\,c} = \frac{\mathcal{E}}{\mathcal{E}_{\rm cr}}\frac{m\,c^2\,T}{\hbar}$$

effective action: $\Gamma(\mathcal{E}, \xi_0) = \sum_{n,l} a_{n,l} \left(\frac{\mathcal{E}}{\mathcal{E}_{\rm cr}}\right)^n \left(\frac{\hbar}{m\,c^2\,T}\right)^l$

exact Borel integral & resummation of derivative expansion: Cangemi/GD/D'Hoker (1995); GD/Hall (1998)

semiclassical imaginary part:

$$\text{Im}\,\Gamma \sim \begin{cases} e^{-\frac{\pi\mathcal{E}_{\rm cr}}{\mathcal{E}}\left(1-\frac{1}{4\xi_0^2}+\dots\right)} \quad , & \xi_0 \gg 1 \quad \text{(non-pert.)} \\ e^{-\frac{2\pi m\,c^2\,T}{\hbar}(1-\xi_0+\dots)} = e^{-\frac{2\pi\mathcal{E}_{\rm cr}}{\mathcal{E}}\,\xi_0(1-\xi_0+\dots)} \quad , & \xi_0 \ll 1 \quad \text{(pert.)} \end{cases}$$

lesson: constant field limit is subtle

(interesting things happen in a localized oscillatory field)

Locally Constant Field Approximation (LCFA)

LCFA: $\mathrm{Im}\,\Gamma \approx \int d^4x\,\mathcal{E}^2(x)\,e^{-\pi\frac{\mathcal{E}_{\rm cr}}{\mathcal{E}(x)}}$ "sometimes works"

the derivative/gradient expansion is divergent

LCFA can fail when $\mathcal{E}(x)$ changes sign; physically due to quantum interference effects, Stokes phenomenon, ...

reliability of LCFA is strongly dependent on structure [shape and character] of the background field:
difference between spatial and temporal localization

in general, weak-field and broad-field limits **do not commute**

- implications for simulations need to be better understood

mathematical methods for non-perturbative effects:
(i) uniform semiclassical approximation: Seger/Balantekin (1996); GD/Hall (1998); GD/Dabrowski (2014)
(ii) *trans-asymptotics* (O. Costin, 1998) and *resurgence*

Non-perturbative (Semiclassical) QED Effective Action

"non-perturbative" means stationary phase approximation to the proper-time (Borel) integral; combined with semiclassical approximation to $\mathrm{tr}(x|U(s)|x)$, when some parameters become large $\Rightarrow$ there are different non-perturbative limits

$$\Gamma[A] = -\int_0^\infty \frac{du}{u} e^{-i\,m^2 u} \int d^4x \int_{x(u)=x(0)=x} \mathcal{D}x\, e^{i\,S[x]}$$

classical "instanton" (saddle) for proper-time path integral (Feynman, 1951):

$$\ddot{x}_\mu = F_{\mu\nu}(x)\,\dot{x}^\nu$$

$$\longrightarrow \quad \Gamma[A] \approx (\text{fluctuations}) e^{i\,S_{\text{saddle}}[A_{\text{classical}}]}$$

Affleck et al (1984): worldline instantons for constant $F_{\mu\nu}$

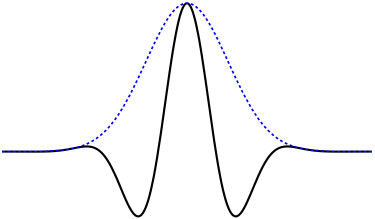
GD/Schubert (1994): worldline instantons for $F_{\mu\nu}(x)$

[suitable for multi-dimensional fields]

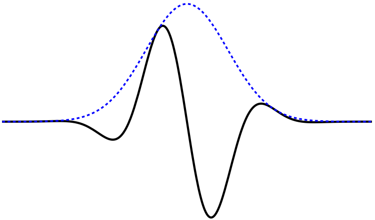
Dumlu/GD (2010): complex saddles for interference

Non-perturbative (Semiclassical) QED Effective Action

quantum interference in “designer” laser pulses $\mathcal{E}(t)$



physics dominated by just one saddle



physics dominated by two equal-weight saddles: non-trivial interference

goal: optimization via (spatio-temporal) pulse design & control

crucial features: the saddle point locations

Non-perturbative (Semiclassical) QED Effective Action

other formulations of vacuum pair production problem:

- scattering picture (Feynman; Pitaevski; ...)
- Bogoliubov (Popov/Marinov; Schützhold et al; ...)
- quantum kinetic equation (Mottola et al; Rau et al; Bloch et al; Blaschke et al; ...)
- Dirac-Heisenberg-Wigner (Bialynicki-Birula et al; Hebenstreit et al; ...)
- space-time instantons (Kim/Page; ...)
- nonequilibrium and real-time classical-statistical ensembles (Gelis et al; Berges et al; ...)
- spectral representation (Fukushima; ...)
- Schrödinger picture (Vachaspati et al; Padmanabhan; ...)
- ...

Some Lessons from Vacuum Pair Production

"Schwinger limit" ($\mathcal{E} \approx \mathcal{E}_{\rm cr}$) is not a sharp limit: required laser intensity can be lowered by several orders of magnitude by clever design of laser pulses; but it is still beyond current laser intensities

laser pulse shaping has a significant effect on the strength of vacuum polarization effects (cf. strong-field ionization)

frequency ω; pulse length τ; carrier phase θ; chirp; pulse sequences; catalysis, ...

these parameters are used routinely to probe interesting physics in strong-field AMO and CM physics

non-equilibrium & back-reaction effects need further work

question:

can similar physical effects be optimized and/or controlled for lepton bunches?

Higher loops: two loop

- Ritus (1975, 1977); Lebedev-Ritus (1984); Fliegner et al (1997): Two-loop: exact expression for constant background field

  double proper-time integral: each lepton propagator has a "proper-time" integral

  renormalization group: strong-field ↔ short-distance

  two-loop emphasizes importance of the “formation region”

- Gies/Karbstein (2016): extra finite contribution at 2-loop

Higher loops: two-loop and beyond

Two-loop (Ritus): exact (double-integral) expression for constant background field

mass renormalization in the imaginary part:

$$e^{-\pi\, m^2/(e\,\mathcal{E})} \to e^{-\pi\, m^2_{\rm ren}/(e\,\mathcal{E})}$$

2-loop pair production in $\mathcal{E}$ field:

$$\mathrm{Im}\,\Gamma \approx e^{-\pi \frac{\mathcal{E}_{\rm cr}}{\mathcal{E}}}\left(1+\pi\,\alpha+\dots\right)$$

Affleck/Alvarez/Manton (1982): weak $\mathcal{E}$ field, all α orders

$$\mathrm{Im}\,\Gamma \approx e^{-\pi \frac{\mathcal{E}_{\rm cr}}{\mathcal{E}}}\, e^{\pi\,\alpha}$$

GD/Schubert (2002): closed 2-loop expression for self-dual field ($\mathcal{E} = \pm i\,\mathcal{B}$):

$$\Gamma^{(2)} = c_1 \left(\Gamma^{(1)}\right)^2 + c_2 \left(\Gamma^{(1)}\right)'$$

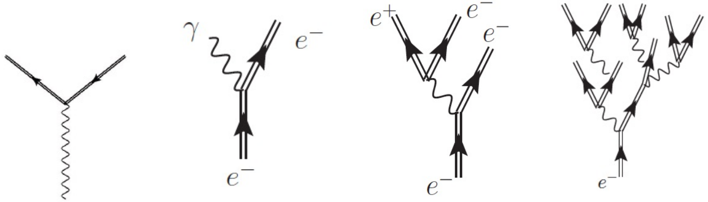
Probe Particles

direct vacuum pair-production is not accessible at available laser intensities

introduce ultra-relativistic probe particles with $p \approx \frac{\mathcal{E}_{\mathrm{cr}}}{\mathcal{E}}\, m\, c$

in rest-frame, electric field is strongly Lorentz-boosted

Breit-Wheeler; nonlinear Compton; trident; cascades; ...



Ritus et al; Baier et al; Milstein et al; Di Piazza et al; Müller et al; Heinzl et al; Marklund et al; Ilderton et al; Hartin et al

...

Furry Picture

Furry (1935): exact states/propagators for leptons; normal QED interaction vertices

(i) constant background field (for e.g., CCF)
(ii) plane-wave field (now non-vanishing result)

new parameter:
"quantum non-linearity parameter" = amplitude of $\mathcal{E}$ field:

in electron rest-frame: $\chi_0 = \frac{e\,\hbar}{m^3\,c^4}\sqrt{-\left(F_{\mu\nu}\,p^\nu\right)^2}$

in $e^\pm$ center-of-mass-frame: $\kappa_0 = \frac{e\,\hbar}{m^3\,c^4}\sqrt{-\left(F_{\mu\nu}\,k^\nu\right)^2}$

mass operator: $M(\mathcal{E}, \xi_0, \chi_0)$

polarization operator: $P(\mathcal{E}, \xi_0, \kappa_0)$

Furry Picture

(i) exact states & propagators in constant field $F_{\mu\nu}$

(ii) Volkov solution in plane-wave: $A_\mu = A\, e_\mu\, \psi(k\cdot x)$

$$\Psi_{pr}(x) = \left[1 + \frac{e\,\hat{k}\cdot\hat{A}}{2\,k\cdot p}\right]\, u_{pr}\, \exp\left[i\,p\cdot x + i\int_0^{k\cdot x}\left(\frac{e\,p\cdot A}{k\cdot p} - \frac{e^2\,A^2}{2\,k\cdot p}\right) d\phi\right]$$

$\Rightarrow$ exact propagators in plane-wave field

first order polarization tensor: $\Pi_{\mu\nu} = -i\,e^2\,\mathrm{tr}\,(\gamma_\mu G\,\gamma_\nu\,G)$

first order mass operator: $M = i\,e^2\,\left(\gamma^\mu G\,\gamma^\nu\,D^{(0)}_{\mu\nu}\right)$

computations often expressed in terms of matrix elements in Volkov states $\Rightarrow$

"non-perturbative" = <u>stationary phase approximation</u>

(in general, there are many competing parameters)

Furry Picture: modified analytic structure

Ritus (1972; review, 1984):

in external field, electron always radiates $\Rightarrow \mathrm{Im}(M) \neq 0$

$\Rightarrow$ analyticity properties of Green's function different from vacuum situation

$\Rightarrow$ Green's function has no singularities on real p^2 axis, but $M(p^2, \chi_0)$ has an essential singularity at $\chi_0 = 0$.

Ritus-Narozhny Conjecture(s)

<u>conjecture 1</u>:

$$\text{ultra-relativistic probe particle in a plane wave}$$
$$\equiv$$
$$\text{ultra-relativistic probe particle in a constant-crossed-field: } \mathcal{E} \perp \mathcal{B}, |\mathcal{E}| = |\mathcal{B}|$$

<u>conjecture 2</u>:

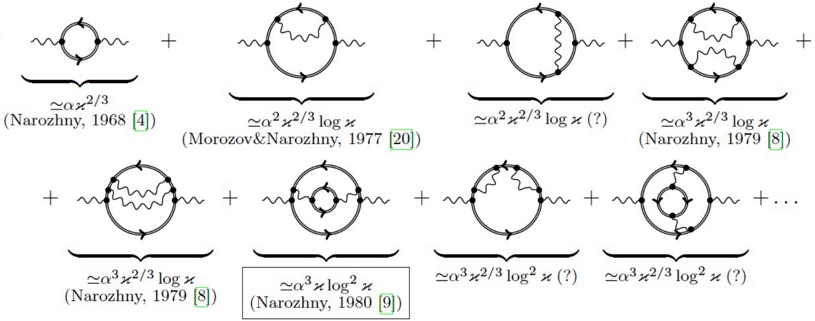
$$\text{in high-energy limit, } \chi_0 \gg 1\text{, QED processes scale like } \alpha\, \chi_0^{2/3}$$

- this differs from the expected logarithmic behavior of QED processes at high energy
- these conjectures involve the locally constant field approximation

Ritus-Narozhny Conjecture(s)

higher loop processes in constant-crossed-field (CCF) have rich structure, again very different from vacuum case:

$$\frac{\mathscr{P}}{m^2} =$$



review, Fedotov (2019)

$\longrightarrow$ conjecture that QED expansion parameter is $\alpha\, \chi_0^{2/3}$ in the large χ_0 regime

Ritus-Narozhny Conjecture(s): High Intensity vs. High Energy

Dinu et al (2016), Di Piazza et al (2018), Ilderton (2019), ...

$$P(\mathcal{E}, \xi_0, \kappa_0) \qquad \text{or} \qquad M(\mathcal{E}, \xi_0, \chi_0)$$

low frequency/CCF limit of a plane wave:

$$\xi_0 \to \infty \quad \text{with} \quad \chi_0 \quad \text{fixed}$$

high energy limit in a plane wave:

$$\chi_0 \to \infty \quad \text{with} \quad \xi_0 \quad \text{fixed}$$

relevant regime governed by magnitude of $\frac{\xi_0^3}{\chi_0}$ (or $\frac{\xi_0^3}{\kappa_0}$)

Ritus-Narozhny Conjecture(s):
High Intensity & High Energy limits do not commute

Di Piazza et al (2018): $P(\mathcal{E}, \xi_0, \kappa_0)$

low frequency/CCF limit of a plane wave:

$$\xi_0 \to \infty \quad \text{with} \quad \kappa_0 \quad \text{fixed}$$

$$\text{Re}(P) \sim \alpha\, \kappa_0^{2/3} \quad , \quad \xi_0 \to \infty \quad , \quad \text{large } \kappa_0$$

$$\text{Im}(P) \sim \alpha\, \kappa_0^{2/3} \quad , \quad \xi_0 \to \infty \quad , \quad \text{large } \kappa_0$$

high energy limit in a plane wave:

$$\kappa_0 \to \infty \quad \text{with} \quad \xi_0 \quad \text{fixed}$$

$$\text{Re}(P) \sim \alpha\, \xi_0^2 \ln^2\left(\frac{\kappa_0}{\xi_0}\right) \quad , \quad \kappa_0 \to \infty$$

$$\text{Im}(P) \sim \alpha\, \xi_0^2 \ln\left(\frac{\kappa_0}{\xi_0}\right) \quad , \quad \kappa_0 \to \infty$$

Ritus-Narozhny Conjecture(s)

detailed scaling behavior (logarithmic versus power-law) is process-dependent, and the limits connecting a plane-wave field and a CCF are subtle

rich area for exploration, especially at higher loops

question: how would one probe these fundamental theoretical issues at a lepton collider, in the large χ_0 regime?

Conclusions

• regime of large quantum non-linearity parameter, χ_0 or κ_0, is *terra incognita*, experimentally and (partly) theoretically

• non-perturbative methods are better-developed for vacuum processes than for processes involving probes & collisions, but several formalisms exist to improve this situation

- what are the most important computations to be done?
- are new methods/approximations needed?
- input from other fields (plasma, laser, astro, nuclear, particle, collider, AMO, CM, ...)
- precise relations between QFT and simulations: feedback needed between theory, simulations and experiment
- for lepton collider, need detailed computations of the effect of beam & bunch design and control