MAX-PLANCK-GESELLSCHAFT

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# Perturbation theory in strong-field QED

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Physics Opportunities at a Lepton Collider
in the Fully Nonperturbative QED
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# Outline

- Introduction
  - Typical electromagnetic scales
  - Sources of strong fields for QED
- General considerations about strong-field QED
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- Furry picture for investigating strong-field QED phenomena
- A quick glance at non-perturbative strong-field QED
- Conclusions

# Typical scales of QED

| | |
|---|---|
| Strength:<br>$\alpha = e^2/4\pi\hbar c = 7.3\times 10^{-3}$<br>(Fine-structure constant) | Energy:<br>$mc^2 = 0.511$ MeV<br>(Electron rest energy) |
| Length:<br>$\lambda_C = \hbar/mc = 3.9\times 10^{-11}$ cm<br>(Compton wavelength) | Field:<br>$E_{cr} = m^2c^3/\hbar\lvert e\rvert = 1.3\times 10^{16}$ V/cm<br>$B_{cr} = m^2c^3/\hbar\lvert e\rvert = 4.4\times 10^{13}$ G<br>(Critical fields of QED) |

# Intensity scale

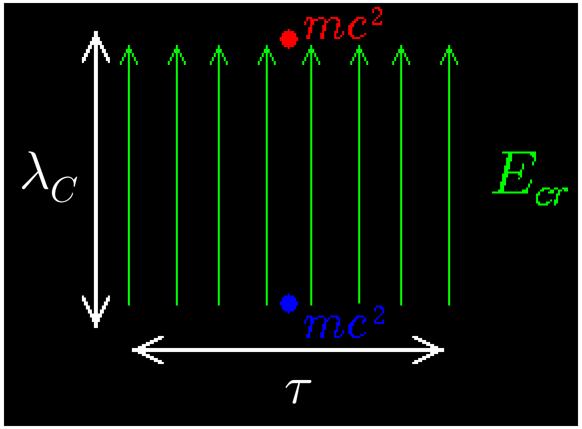
$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3\times 10^{16}\ \text{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4\times 10^{13}\ \text{G}$$

$$\Rightarrow\quad I_{cr} = cE_{cr}^2 = 4.6\times 10^{29}\ \text{W/cm}^2$$

# Physical meaning of the critical fields

$$|e|E_{cr} \times \frac{\hbar}{mc} = mc^2$$

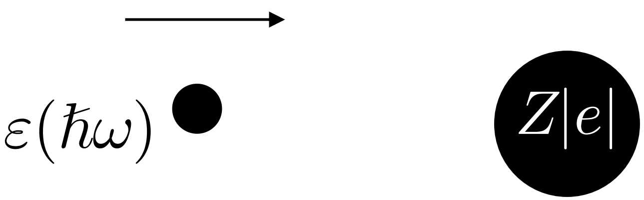



- Vacuum instability and electromagnetic cascades (Bell et al. 2008, Bulanov et al. 2010, Fedotov et al. 2010)
- The interaction energy of a Bohr magneton with a magnetic field of the order of $B_{cr}$ is of the order of the electron rest energy
- In the presence of background electromagnetic fields of the order of the critical ones a new regime of QED, the strong-field QED regime, opens:

1. where the properties of the vacuum are substantially altered by the fields
2. where a tight interplay unavoidably exists between collective (plasma-like) and quantum effects

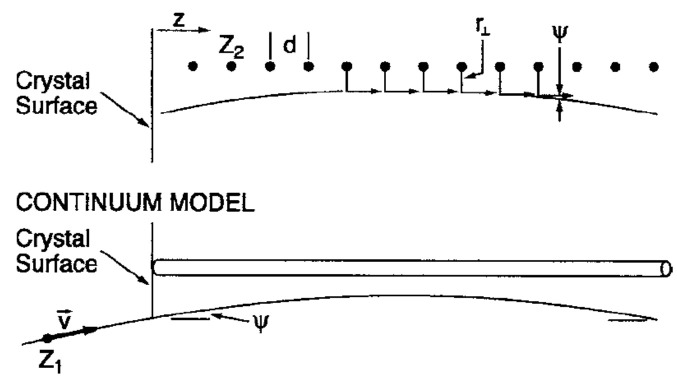
# Sources of strong electromagnetic fields

- Highly-charged ions (Bethe and Heitler 1934, Bethe and Maximon 1954)

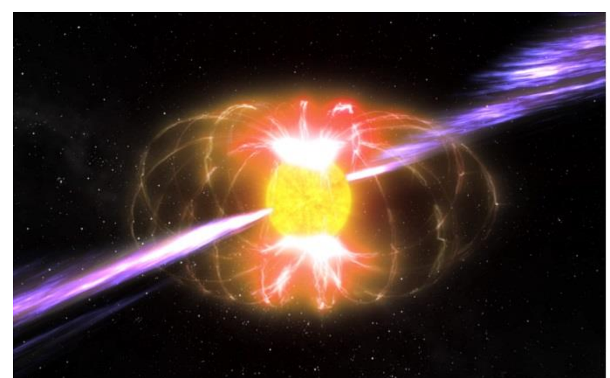



High-order nonlinear QED effects (Coulomb corrections) only depend on the parameter $Z\alpha$ ($\alpha$=1/137)

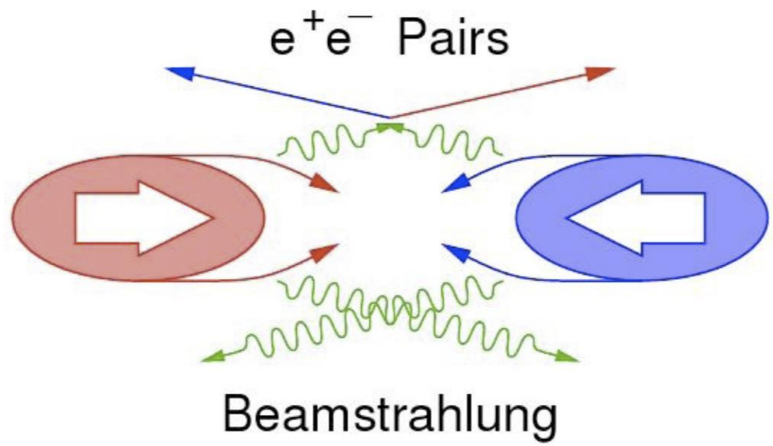
- Crystals and channeling (Uggerhøj 2005): ultrarelativistic charged particles interact coherently with the atoms aligned in the crystal




- Magnetars (Turolla et al. 2015): rotating neutron stars whose surrounding magnetic fields are estimated to even exceeding the critical one

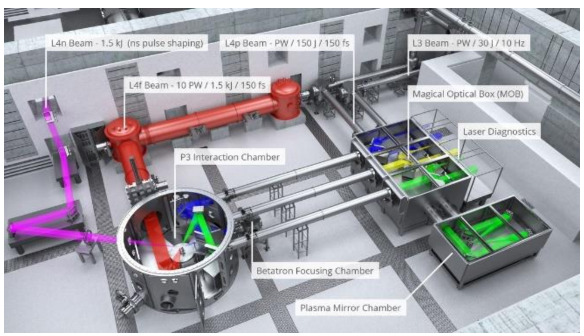


- Ultrarelativistic electron-positron bunches (Chen 1987): in collisions between $e^+$-$e^-$ beams as those presumably occurring in future linear colliders, strong-field QED effects may limit the performances of such colliders (beamstrahlung)




- Intense lasers (Di Piazza et al. 2012):

ELI Beamlines (Czech Republic)




XCELS (Russia)

ELI NP (Romania)

# Sources of strong electromagnetic fields

| | Electric field (V/cm) | Electric field ($E_{cr}$) | Variation length scale (nm) |
|---|---|---|---|
| Highly-charged ions (hydrogen-like Lead ($Z$=82) in the ground state) | $10^{15}$ | $10^{-1}$ | 0.1 |
| Strong optical lasers (RAL, UK) | $10^{12}$ | $10^{-4}$ | 1000 |
| Bunch-bunch collision (FACET-II) | $10^{10}$ | $10^{-6}$ | 1000 |
| Crystals at channeling (Si $\langle 111 \rangle$) | $10^{10}$ | $10^{-6}$ | 0.1 |

# Sources of ultrarelativistic electrons

| | Energy (GeV) | Beam duration (fs) | Number of electrons |
|---|---|---|---|
| Conventional accelerators (SLC, SLAC) | 50 | $3\times10^{3}$ | $4\times10^{10}$ |
| Laser-plasma accelerators (BELLA, LBNL) | 8 | 40 | $3\times10^{9}$ |
| CERN tertiary beam (NA63) | 180 | 1000/minute | |

# Electromagnetic field as classical field

- Following Bohr, an electromagnetic field can be treated as a classical field if the occupation numbers $n_{\boldsymbol{k},\lambda}$ corresponding to the operators $N_{\boldsymbol{k},\lambda}=c^{\dagger}_{\boldsymbol{k},\lambda}c_{\boldsymbol{k},\lambda}$ are large. However, if all $n_{\boldsymbol{k},\lambda}$ are large, the energy of the field would be infinite (Landau and Lifshitz 1982)
- If the field is measured during a time $\Delta t$, angular frequencies larger than $\omega_0=1/\Delta t$ cannot be resolved
- Require that $n_{\boldsymbol{k},\lambda}\gg 1$ for $\omega=c|\boldsymbol{k}|<\omega_0$
- Typical occupation number $n$ in terms of the fields $(E, B)$

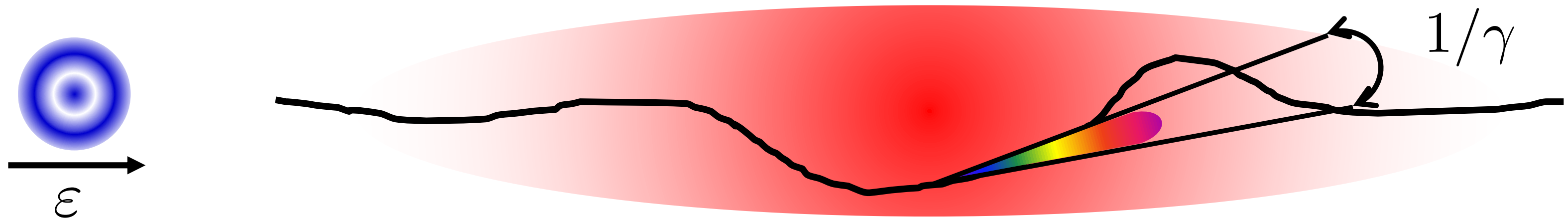
$$n \sim \frac{\text{Total field energy}}{\text{Typical number of states} \times \text{Typical states' energy}}$$
$$\sim \frac{\frac{1}{2}(E^2+B^2)V}{\frac{V}{(2\pi\hbar)^3}\frac{4}{3}\pi\left(\frac{\hbar\omega_0}{c}\right)^3 \times (\hbar\omega_0)} = 1.2\times 10^{-3}\frac{I[\mathrm{W/cm^2}]}{(\hbar\omega_0[\mathrm{eV}])^4}$$

- The condition $n\gg 1$ is easily fulfilled for available optical lasers
- The classical spacetime evolution of the field should not be altered during the quantum process

# General considerations

Units with $\hbar = c = 1$

- An electron with initial energy $\varepsilon \gg m$ enters a region where a strong electromagnetic field is present




- Quantum effects related to the motion of the electron in the field are negligible in the ultrarelativistic regime (Baier et al. 1989):
  1. $|[p^\mu, p^\nu]| = |eF^{\mu\nu}| \ll \varepsilon^2$, where $p^\mu = P^\mu - eA^\mu$ is the kinetic four-momentum of the electron
  2. $|\nabla \lambda_{DB}| = (|\nabla \lambda_{DB}|/\lambda_{DB})\lambda_{DB} \ll 1$, where $\lambda_{DB} = 2\pi/p$ is the De Broglie wavelength of the electron (WKB regime)

- From relativistic kinematics the electron instantaneously emits along its velocity within a cone of aperture $m/\varepsilon = 1/\gamma \ll 1$:

$$k_\parallel = \gamma(k_{\parallel,\mathrm{irf}} + \beta\omega_{\mathrm{irf}})$$
$$\boldsymbol{k}_\perp = \boldsymbol{k}_{\perp,\mathrm{irf}}$$

- A relevant quantity in strong-field physics is the ratio between the instantaneous emission angle $\theta_i \sim 1/\gamma$ and the total opening angle $\theta_T \sim |\boldsymbol{p}_{\perp,\max}|/\varepsilon$ ($\perp$ with respect to the average direction of motion)

$$\xi = \theta_T/\theta_i \sim |\boldsymbol{p}_{\perp,\max}|/m$$

- Dipole or perturbative regime ($\xi \ll 1$) and synchrotron or local-constant-field regime ($\xi \gg 1$)
- Another parameter controls the importance of quantum effects (photon recoil, $e^+$-$e^-$ pair production): the strength of the electromagnetic field in the instantaneous rest frame of the electron in units of the critical field $F_{cr}$ (Ritus 1985)

$$\chi = \frac{F_{\text{irf}}}{F_{cr}} \sim \gamma \frac{F}{F_{cr}}$$

- The Lorentz-invariant electric and magnetic fields

$$\mathcal{E} = \sqrt{\sqrt{\mathcal{F}^2 + \mathcal{G}^2} - \mathcal{F}} \qquad \mathcal{B} = \sqrt{\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F}}$$

with $\mathcal{F}=-(E^2-B^2)/2$ and $\mathcal{G}=-\boldsymbol{E}\cdot\boldsymbol{B}$, determine the dielectric properties of the vacuum and its stability for background fields slowly varying on a Compton wavelength

# Furry picture in a strong background field

- The Lagrangian density of QED in the presence of a background field $F_B^{\mu\nu} = \partial^\mu A_B^\nu - \partial^\nu A_B^\mu$ produced by a four-current $J_B^\mu$ is given by

$$\mathscr{L}_{QED} = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu - eA_{B,\mu}) - m]\psi + \mathscr{L}_B - J_B^\mu(A_\mu + A_{B,\mu}) - \frac{1}{4}(F_{\mu\nu} + F_{B,\mu\nu})(F^{\mu\nu} + F_B^{\mu\nu})$$

- The background field and four-current are given functions fulfilling Maxwell's equations $\partial_\mu F_B^{\mu\nu} = J_B^\nu$, and we can drop the "constant" terms

$$\mathscr{L}_{QED} \sim \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu - eA_{B,\mu}) - m]\psi - J_B^\mu A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu\nu}F_B^{\mu\nu}$$

- By integrating by parts the term

$$F^{\mu\nu}F_{B,\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu)F_{B,\mu\nu} = \partial^\mu(A^\nu F_{B,\mu\nu}) - A^\nu(\partial^\mu F_{B,\mu\nu}) - \partial^\nu(A^\mu F_{B,\mu\nu}) + A^\mu(\partial^\nu F_{B,\mu\nu})$$
$$= \partial^\mu(A^\nu F_{B,\mu\nu}) - \partial^\nu(A^\mu F_{B,\mu\nu}) - 2A^\nu J_{B,\nu}$$

we obtain (Furry 1951)

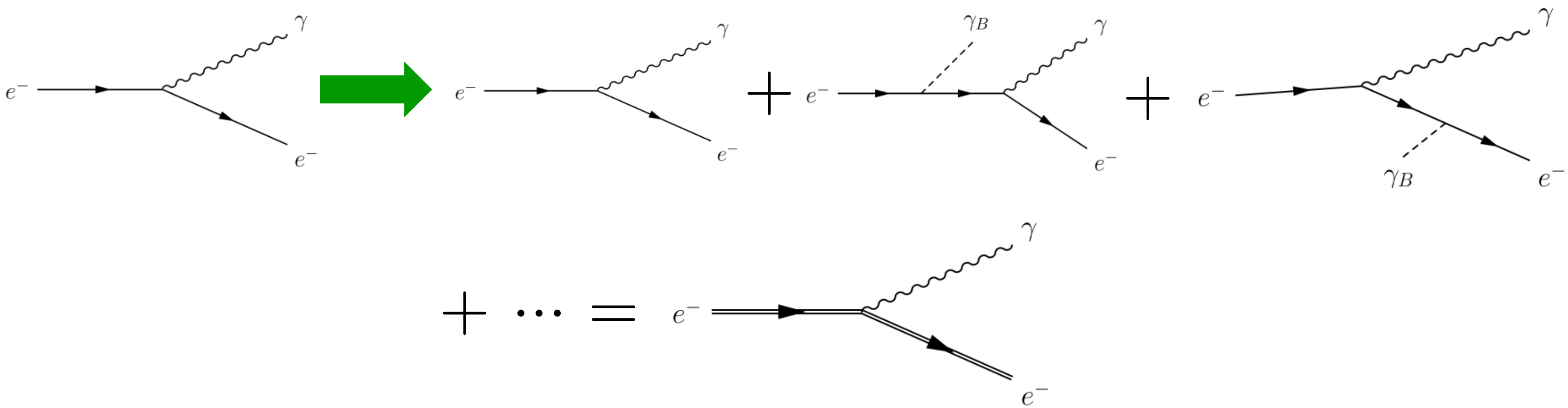
$$\mathscr{L}_{QED} \sim \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu - eA_{B,\mu}) - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- The effect of the external field is to give rise to an additional vertex corresponding to the interaction term $\mathscr{L}_{i,B} = -e\bar{\psi}\gamma^\mu\psi A_{B,\mu}$

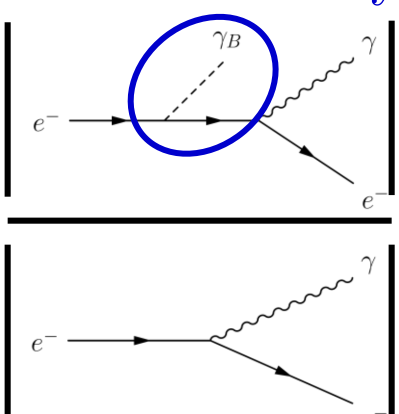
- The Lagrangian density we employ in strong-field QED is

$$\mathscr{L}_{SFQED} = \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu} - eA_{B,\mu}) - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- The effect of the external field on a QED process




- The contribution of the external field has to be taken into account exactly in the calculations for




$$\sim \frac{|e|A_B}{m} \sim 1$$

The expression of the quantity $\rho = |e|A_B/m$ depends on the external field:

1. HCI: $\rho = Z\alpha/\lambda_C m = Z\alpha$
2. PW: $\rho = |e|E_0/\omega_0 m = \xi$

- If the background field is strong ($\rho \gtrsim 1$), the spinor field is quantized in the presence of the background field and only the interaction between the spinor and the radiation field is treated perturbatively

$$\mathscr{L}_{SFQED} = \mathscr{L}_e + \mathscr{L}_\gamma + \mathscr{L}_i$$

$$\mathscr{L}_e = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi$$

$$\mathscr{L}_\gamma = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathscr{L}_i = -e\bar{\psi}\gamma^\mu\psi A_\mu$$

- The quantization of the spinor field in the presence of the background field implies the ability of solving analytically the "dressed" Dirac equation $[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]\psi=0$:
  1. Find the positive-energy and negative-energy solutions that asymptotically reduce to "free" plane waves for $t \to \pm\infty$ (dressed out- and in-states)
  2. Find the dressed Feynman propagator by solving the equation $[\gamma^\mu(i\partial_\mu - eA_{B,\mu}) - m]G(x,y)=\delta^4(x-y)$
  3. Write the Feynman diagrams of the process at hand
  4. Calculate the total amplitude and then the cross section (or the rate) using "dressed" states and propagators

# Some remarks

- New types of diagrams with respect to the vacuum theory turn out not to vanish (single-vertex diagrams, tadpole diagrams)
- Depending on the structure of the external field only some components of the energy-momentum four-vector are conserved
- Exact analytical solutions of the dressed Dirac equation are available for few physically relevant background electromagnetic fields (plane waves, constant fields, Coulomb field)

# Examples

- Basic strong-field QED processes (nonlinear photon emission and nonlinear electron-positron photoproduction) have been studied in magnetic field, plane-wave field
- Two-loops radiative corrections are being investigated in the case of highly-charged ions
- Second-order processes (nonlinear double Compton scattering, trident process) have been studied in the case of a plane wave

# High-energy behavior of strong-field QED

- Radiative corrections in vacuum QED scale logarithmically with the energy scale
- Ritus and Narozhny (RN) observed that radiative corrections in a constant-crossed field (CCF) scale as the 2/3-power of the energy scale at $\chi \gg 1$ (Ritus 1970, Narozhny 1979, 1980, Fedotov 2017)
- RN formulated the conjecture that at high energy scales such that $\alpha\chi^{2/3} \sim 1$ the perturbative approach to QED would break down
- The importance of the RN conjecture relies on the fact that at sufficiently high value of $\xi$ the results in an arbitrary plane wave reduce to those in a locally constant-crossed field
- What is the reason of such a different behavior between vacuum QED and strong-field QED?
- In PRD **99**, 076004 (2019) (arXiv:1812.08673) we investigated the dependence of the leading-order polarization operator (mass operator) in a generic plane wave on the parameters $\xi$ and $\theta=(k_0k)/m^2$ ($\xi$ and $\eta=(k_0p)/m^2$). Note: $\kappa=\theta\xi$ and $\chi=\eta\xi$.
- Consider the mass operator only

- Constant-crossed-field (CCF) limit vs high-energy (HE) limit

| CCF limit | HE limit |
|---|---|
| $\xi = \lvert e\rvert E_0/m\omega_0 \to \infty$<br>$\eta = (k_0 p)/m^2 \to 0$<br>such that $\chi = \eta\xi$ is fixed | $\eta = (k_0 p)/m^2 \to \infty$<br>$\chi = (k_0 p)E_0/m\omega_0 E_{cr} \to \infty$<br>such that $\xi = \chi/\eta$ is fixed |

- The parameter $r = \xi^3/\chi$ is large (small) in the CCF (HE) limit
- In doing the limit $\chi \to \infty$ within the CCF limit we should remember that the quantity $r = \xi^3/\chi$ has to be large for the CCF limit to be applicable
- Analogously as in vacuum, in the HE limit the asymptotic expression of the mass operator was found to feature a (double) logarithmic dependence on $\eta$ (PRD **99**, 076004 (2019))
- These results do not preclude the possibility of testing experimentally the RN conjecture (Blackburn et al. 2018, Yakimenko et al. 2018, and Baumann et al. 2018)
- Radiative corrections in vacuum for an on-shell incoming particle vanish and the logarithmic dependence on the energy scale there is via the electron "virtuality" $p^2/m^2$

# Conclusions

- There is an increasing interest in studying QED processes in the presence of strong background electromagnetic fields
- The theoretical investigation of such processes predominantly relies on a semi-perturbative approach (Furry picture) where
  - The interaction between charged particles (electrons and positrons) and the background field is taken into account exactly in the calculations
  - The interaction between charged particles and the radiation field is treated perturbatively
- It has been conjectured that the semi-perturbative approach should break down at $\alpha\chi^{2/3} \sim 1$ when a fully non-perturbative regime of QED should be entered
- Is strong-field QED at $\alpha\chi^{2/3} \sim 1$ a strongly coupled theory as QCD?