Photon Emission Probability Beyond Tree Level - Possible Approaches and Review of the Literature

Workshop: Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime

Sebastian Meuren
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(Semi-) Classical Electrodynamics
14.6 Frequency Spectrum of Radiation Emitted by a Relativistic Charged Particle in Instantaneously Circular Motion

The treatment of synchrotron radiation presented here is completely classical, but the language of photons can be used, if desired. The number of photons per unit frequency interval is obtained by dividing the intensity distribution (14.91) [or (14.79)] by $\hbar \omega$. Then the photon frequency distribution is

$$\frac{dN}{dy} = \frac{I}{\hbar \omega_c} \cdot \frac{9\sqrt{3}}{8\pi} \int_y^\infty K_{5/3}(x) \, dx$$  \hspace{1cm} (14.93)

where $y = \omega/\omega_c$ and $I = 4\pi e^2 \gamma^4/3\rho$ is the total energy radiated per revolution. Integration over frequency gives the mean number of photons emitted per revolution per particle,

$$N = \frac{5\pi}{\sqrt{3}} \gamma \alpha$$  \hspace{1cm} (14.94)

where $\alpha$ is the fine structure constant. The mean energy per photon is $I/N$:

$$\langle \hbar \omega \rangle = \frac{8}{15\sqrt{3}} \hbar \omega_c$$  \hspace{1cm} (14.95)

- Charge in background field $\rightarrow$ acceleration $\rightarrow$ charge emits radiation
- Classical electrodynamics predicts emission probability & spectrum
- Question: when do quantum corrections become important?
Quantum effects ($\chi \sim 1$): a first hint from classical electrodynamics

- QED critical field $E_{cr} = m^2 c^3 / (e \hbar) \approx 10^{18}$ V/m [$B_{cr} = m^2 c^2 / (e \hbar) \approx 10^9$ T]: appears already in the classical synchrotron emission spectrum if we assume that photons are quantized ($\hbar$)

- Emission probability decreases exponentially for $\omega > \omega_c$, where $\hbar \omega_c = (2/3) \chi \epsilon$
  - $\epsilon$: electron/positron energy; $\chi$: “quantum parameter”; static magnetic field $B$: $\chi = \gamma B / B_{cr}$

- The result is “nonperturbative” in the charge: $dP/d\omega \sim \sqrt{\omega / \omega_c} \exp(-\omega / \omega_c) \sim \exp(-1/e)$

- If $\chi \gtrsim 1$ we expect breakdown of classical physics (energy conservation is violated)
Quantum effects ($\chi \sim 1$): heuristic model based on classical result

\[ P(\chi) = \frac{dI_{\text{rad}}}{dt} = \int_0^\infty d\omega \frac{dI_{\text{rad}}}{d\omega dt}(\chi, \omega) \]

\[ \frac{dI_{\text{rad}}^{\text{class}}}{d\omega dt}(\chi, \omega) = -\alpha \frac{m^2}{\epsilon} u \left[ \int_2^\infty dv \ Ai(v) + 2 \frac{Ai'(z)}{z} \right] \]

Classical result for the total radiated power $P$ (energy per unit time) and for the spectrum ($dP/d\omega$)

- Classically, we obtain $u = \omega/\epsilon$ ($\epsilon$: energy of incoming electron), $z = (u/\chi)^{2/3}$
- Heuristically, we can apply the replacement $\epsilon \rightarrow \epsilon - \omega$ and obtain $u = \omega/(\epsilon - \omega)$
- Due to the divergence the probability is zero at $\omega = \epsilon$ (energy conservation)

\[ \frac{dI_{\text{rad}}}{d\omega dt}(\chi, \omega) = -\alpha \frac{m^2}{\epsilon} u \left\{ \int_2^\infty dv \ Ai(v) + \frac{Ai'(z)}{z} \left[ 2 + \frac{u^2}{1 + u} \right] \right\} \]

Result of the full quantum calculation

Plot: total emitted power $P$ [$P_0 = (mc^2)^2/\hbar$]; dashed: classical, dotted: classical + recoil, solid: quantum
(Strong-Field) Quantum Electrodynamics
Photon emission probability: perturbative QED perspective

The Klein-Nishina Formula

The rest of the computation of the Compton scattering cross section is straightforward, although it helps to be somewhat organized. We want to average the squared amplitude over the initial electron and photon polarizations, and sum over the final electron and photon polarizations. Starting with expression (5.74) for $\mathcal{M}$, we find

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4} g_{\mu\rho} g_{\nu\sigma} \cdot \text{tr} \left\{ (p'+m) \left[ \frac{\gamma^\mu \gamma^\rho + 2 \gamma^\mu p^\rho}{2p \cdot k} + \frac{\gamma^\nu \gamma^\sigma - 2 \gamma^\sigma p^\mu}{2p \cdot k'} \right] \right\}$$

$$\equiv \frac{e^4}{4} \left[ \frac{\text{I}}{(2p \cdot k)^2} + \frac{\text{II}}{(2p \cdot k)(2p \cdot k')} + \frac{\text{III}}{(2p \cdot k')(2p \cdot k)} + \frac{\text{IV}}{(2p \cdot k')^2} \right], \quad \text{(5.81)}$$

- Linear Compton scattering: textbook problem of perturbative QED
- Question: when do nonlinear effects become important?
Photon emission in oscillatory field: nonlinear effects

Interaction of Intense Laser Beams with Electrons

LOWELL S. BROWN† AND T. W. B. KIBBLE
Department of Physics, Imperial College, London, England
(Received 15 July 1963; revised manuscript received 16 September 1963)

We may now evaluate this integral in the laboratory frame in which

\[ \mathbf{p} \cdot \mathbf{k} = m\omega, \quad \mathbf{p} \cdot \mathbf{k} = m\omega', \quad \mathbf{k} \cdot \mathbf{k}' = 2\omega\omega' \sin^2 \frac{\theta}{2}, \]

where \( \omega \) and \( \omega' \) are the energies corresponding to \( \mathbf{k} \) and \( \mathbf{k}' \) and \( \theta \) is the scattering angle of the photon. The vanishing of the argument of the \( \delta \) function gives the relation between the incident and scattered frequencies,

\[ \omega' = \omega \frac{m\omega}{m + (2\omega + m\nu^2) \sin^2 \frac{\theta}{2}}. \quad (3.34) \]
Photon emission in constant field: quantum synchrotron radiation

A. I. Nikishov and V. I. Ritus

\[
\frac{dI_{\text{rad}}}{d\omega dt}(\chi, \omega) = -\alpha \frac{m^2}{\epsilon} \frac{u}{1+u} \left\{ \int_{z}^{\infty} dv \, A_i(v) + \frac{A'_i(z)}{z} \left[ 2 + \frac{u^2}{(1+u)} \right] \right\}
\]

- Total radiated power \( P \sim \alpha (c/l_f) \hbar \omega \sim \chi^{2/3} \) suppressed: typical photon energy \( \hbar \omega \sim \varepsilon \) vs. \( \varepsilon \chi \); formation length: \( l_f \sim 1/\chi^{2/3} \) vs. \( 1/\chi \)

- Due to large recoil, probing \( \chi \gg 1 \) becomes impossible if radiation probability exceeds unity

S. Meuren, Workshop Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime, August 7-9, 2019
Photon emission probability: formation length

Nuclear Physics B328 (1989) 387–405
North-Holland, Amsterdam

QUANTUM RADIATION THEORY IN INHOMOGENEOUS EXTERNAL FIELDS

V.N. BAIER, V.M. KATKOV and V.M. STRAKHOVENKO

Institute of Nuclear Physics, Novosibirsk, 630090, USSR

\[ l_c = \frac{1}{|b|} \left( 1 + \frac{\chi}{u} \right)^{1/3} = \frac{l_0}{\chi} \left( 1 + \frac{\chi}{u} \right)^{1/3}, \quad u = \omega / \epsilon', \]

\[ l_0 = \gamma \lambda_C; \quad \lambda_C = 1/m \text{ is the Compton wavelength of the electron.} \]

- So far we considered a purely constant field
- In “inhomogeneous fields” “Keldysh parameter”: $\sigma/l_f$ $\sigma$: length field is approx. const.; $l_f$: formation length
- Laser field ($\sigma \sim \lambda$): $\sigma/l_f \sim (1/a_0) \left( \frac{u^{1/3}}{\sqrt[3]{\chi}} \right)$ for $\chi \gg 1$
  $a_0$: classical intensity parameter $a_0 \sim \chi m^2 / (\epsilon \omega)$

To test the Ritus-Narozhny conjecture ($\chi \gg 1$ regime) we must ensure LCFA validity
(more details in talk by A. Ilderton)

• Successor of the seminal E-144: 
  \( \approx 50 \text{ GeV}, I \approx 10^{18} \text{ W/cm}^2: a_0 < 1, \chi < 1 \)

• “Probing SFQED at FACET-II” aka E-320: 
  \( \approx 13 \text{ GeV}, I \approx 10^{20} \text{ W/cm}^2: a_0 \approx 7, \chi \approx 1 \)

• First laser measurement in the strong-field quantum regime \( (a_0 \gg 1, \chi > 1) \)

• Measuring the photon formation region: strong-field \( (a_0 \gg 1) \) LCFA breakdown

→ Main talk by D. Reis tomorrow
Formation length of Bremsstrahlung also depends on emitted photon energy:

\[ l_f = \frac{2E(E - \hbar \omega)}{m^2c^3 \omega} \]

LPM effect: bremsstrahlung suppression if the formation length becomes comparable to mean free path (SLAC E-146)

Interference between radiation of two adjacent foils if formation length \( \sim \) gap
Leading-order radiative corrections
Photon emission probability: what have we neglect so far?

- So far we took only the (classical) background field exactly into account:

  \[
  = + + + \ldots
  \]

- We neglected the **emission of multiple photons**:
  - Mitigation (in principle) via short interaction time
  - Advantage of beam-beam collision: extremely short interaction times possible (10 nm \(\cong\) 33 as)
  - More details: **talk by G. Torgrimsson**

- We neglected **radiative corrections**:
  - In perturbative quantum electrodynamics they are usually small \((\alpha/\pi \approx 0.002)\)
  - Ritus-Narozhny conjecture: true parameter \(\alpha \chi^{2/3}\) (**talk by A. Fedotov/A. Mironov**)

\[
+ + + + \ldots
\]
Photon mass: leading-order (1-loop) result

\[ -\partial^2 \Phi^\mu(x) = \int d^4y \mathcal{P}^{\mu\nu}(x, y) \Phi_\nu(y) \]

Diagrammatic representation of the “exact photon wave function”

\[ \begin{array}{c}
\text{Quantum corrections dynamically induce a photon mass (polarization dependent)} \\
\text{Leading-order (one-loop) result, } \chi \gg 1 \text{ limit, constant field approximation}
\end{array} \]

- The asymptotic scaling is independent of m (vacuum lepton mass; \( \chi \sim 1/m^3 \))
- Polarization dependence: “vacuum birefringence”, imaginary part: pair production

Measuring photon mass: vacuum birefringence

- Photon mass is polarization dependent
- Polarization rotates in background field
- Relevant for **astrophysics** (Magnetars): more details: talk by P. Meszaros tomorrow

Gamma + strong optical laser proposal (strong field regime)
Fermionic radiative corrections: leading-order (1-loop) result

\[
\begin{align*}
\text{Diagrammatic representation of the “exact fermionic wave function”} \\
\[i\partial_t + eA(x) - m\] \Psi(x) &= \int d^4y \ M(x, y) \Psi(y)
\end{align*}
\]

Quantum corrections dynamically change the fermionic mass (spin dependent)

\[
m^2_\pm - m^2 \approx \alpha \chi^{2/3} m^2 \frac{14}{9} \frac{\Gamma(2/3)}{3^{1/3} \sqrt{3}} (1 - i\sqrt{3}) \pm \alpha \chi^{1/3} m^2 \frac{1}{9} \frac{\Gamma(1/3)}{3^{2/3} \sqrt{3}} (1 + i\sqrt{3})
\]

Leading-order (one-loop) result, \(\chi \gg 1\) limit, constant field approximation

V. I. Ritus, Ann. Phys. 69, 555–582 (1972)
Measuring strong-field fermionic radiative corrections

\[ \Psi_{p,\sigma}^E(x) = \left[ 1 + \frac{e}{k \hbar} \gamma f \gamma \psi(kx) - \frac{i e}{4 \xi k \hbar} \gamma f \gamma \mu_B(\phi) \right] e^{i S_{p,\sigma}^E(x)} u_{p,\sigma}. \]

• **Two main observables:** **spin dynamic** changes w.r.t. semi-classical BMT equation

• **“Survival probability”** (probability for zero-photon emission) linked to total radiation probability via optical theorem:

• Recently, this was called ”Quantum Quenching of Radiation Losses”
Fermionic mass correction: intuitive picture

- The Volkov wave function: “semiclassic”: $\Psi(x) \sim \exp[iS(x)]; S(x)$ obeys Hamilton-Jacobi:

$$S(x) = -p_0 x + \int_{-\infty}^{kx} d\phi' \left[ \frac{e p_0 A(\phi')}{k p_0} + \frac{e^2 A^2(\phi')}{2 k p_0} \right] [e A^\mu(x) - \partial^\mu S(x)] [e A^\mu(x) - \partial^\mu S(x)] = m^2$$

- Only phase differences are important in quantum mechanics:

$$\Delta S = S(y) - S(x) = \langle p - e A \rangle^\mu (x, y) = \frac{1}{\phi_y - \phi_x} \int_{\phi_x}^{\phi_y} d\phi' \left[ p^\mu(\phi') - e A^\mu(\phi') \right].$$

- Here $p^\mu(\phi)$ is the classical four-momentum, $\phi_x = kx$, $\phi_y = ky$ denotes the laser phases

$$p^\mu(\phi) - e A^\mu(\phi) = p^\mu_0 - k^\mu \left[ \frac{e p_0 A(\phi)}{k p_0} + \frac{e^2 A^2(\phi)}{2 k p_0} \right]$$

- Question: which is the “characteristic length scale” for averaging?

$\rightarrow$ for an oscillatory field with large formation region (!) we set $\phi_y = 2\pi + \phi_x$ and obtain Brown-Kibble-Nikishov-Ritus (purely classical effect)

$\rightarrow$ strong field: small formation region (uncertainty principle): quantum mass correction

$$\langle p - e A \rangle^\mu = p^\mu_0 + \frac{M^2_{\text{osc}}}{2 k p_0} k^\mu,$$

Depending on the field strength we get classical or quantum mass corrections

$$\langle p - e A \rangle^\mu = p^\mu(\phi_z) + \frac{M^2_{\text{ccf}}}{2 k p_0} k^\mu, \quad M^2_{\text{ccf}} \sim m^2 \chi^{2/3}.$$
Fully Nonperturbative QED
Heuristic estimate of nonperturbative corrections

- Do loop corrections suppress the radiation probability further for $\chi \gg 1$?

- Qualitative (naive) argument: fermion mass increases: $m^2 \rightarrow m^* = m^2(1 + 0.84\alpha\chi^{2/3})$
  → effective “critical field” should be higher

- We introduce $\chi^* = \chi^{2/3} (m/m^*)^2$ and heuristically apply the replacement $\chi \rightarrow \chi^*$ in rad. probability


S. Meuren, Workshop Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime, August 7-9, 2019
Vertex correction: “controversial results/claims”

Vertex function of an electron in a constant electromagnetic field

D. A. Morozov, N. B. Narozhnyǐ, and V. I. Ritus

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR
(Submitted 5 December 1980)

Of still greater interest is the qualitative behavior of the vertex function $V^{(3)}_{\mu}$ at large values of the parameters (56). In this case the important values in the expression (54) are $\omega \sim \chi^{-2/3}$ and $\Phi(\epsilon) \sim 1$, and we have the relation

$$V^{(3)}_{\mu} \sim \alpha \chi^{2/3} V^{(1)}_{\mu}.$$  

(59)

Dynamical Chiral Symmetry Breaking in QED in a Magnetic Field: Toward Exact Results

V. P. Gusynin,¹ V. A. Miransky,¹,² and I. A. Shovkovy³,*

Our aim is to show that there exists a gauge in which the approximation with a bare vertex,

$$\Gamma^{\mu}(x, y, z) = \gamma^{\mu} \delta(x - y)\delta(x - z),$$  

(10)

is reliable for the description of spontaneous chiral symmetry breaking in a magnetic field.

see talk by I. Shovkovy tomorrow
Towards fully nonperturbative calculations

- Calculation by A. Fedotov/A. Mironov is a very promising start, has to be extended (resummation of certain diagrams, identification of relevant kinematic ranges)
- Exact propagators and vertex obey exact relations (Schwinger-Dyson equations)
- For electron in lowest Landau level and supercritical fields a truncation scheme was applied, which facilitates fully nonperturbative calculations (see talk by I. Shovkovy)

\[ \mathcal{M}(p) = G^{-1}(p) - \mathcal{G}^{-1}(p) = -ie^2 \int \gamma^\nu \mathcal{G}(p + k) \Gamma^\mu(p + k, p; k) \cdot \mathcal{D}_{\mu\nu}(k) \frac{d^4k}{(2\pi)^4}. \]

\[ \frac{\mathcal{P}_{\mu\nu}(k)}{4\pi} = D^{-1}_{\mu\nu}(k) - \mathcal{D}^{-1}_{\mu\nu}(k) = ie^2 \text{tr} \int \gamma_\mu \mathcal{G}(p + k) \Gamma_\nu(p + k, p; k) \mathcal{G}(p) \frac{d^4p}{(2\pi)^4}; \]
Thank you for your attention