Potentially Relevant Techniques from QCD/Lattice

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Physics Opportunities at a Lepton Collider in the Fully Nonperturbative QED Regime

SLAC

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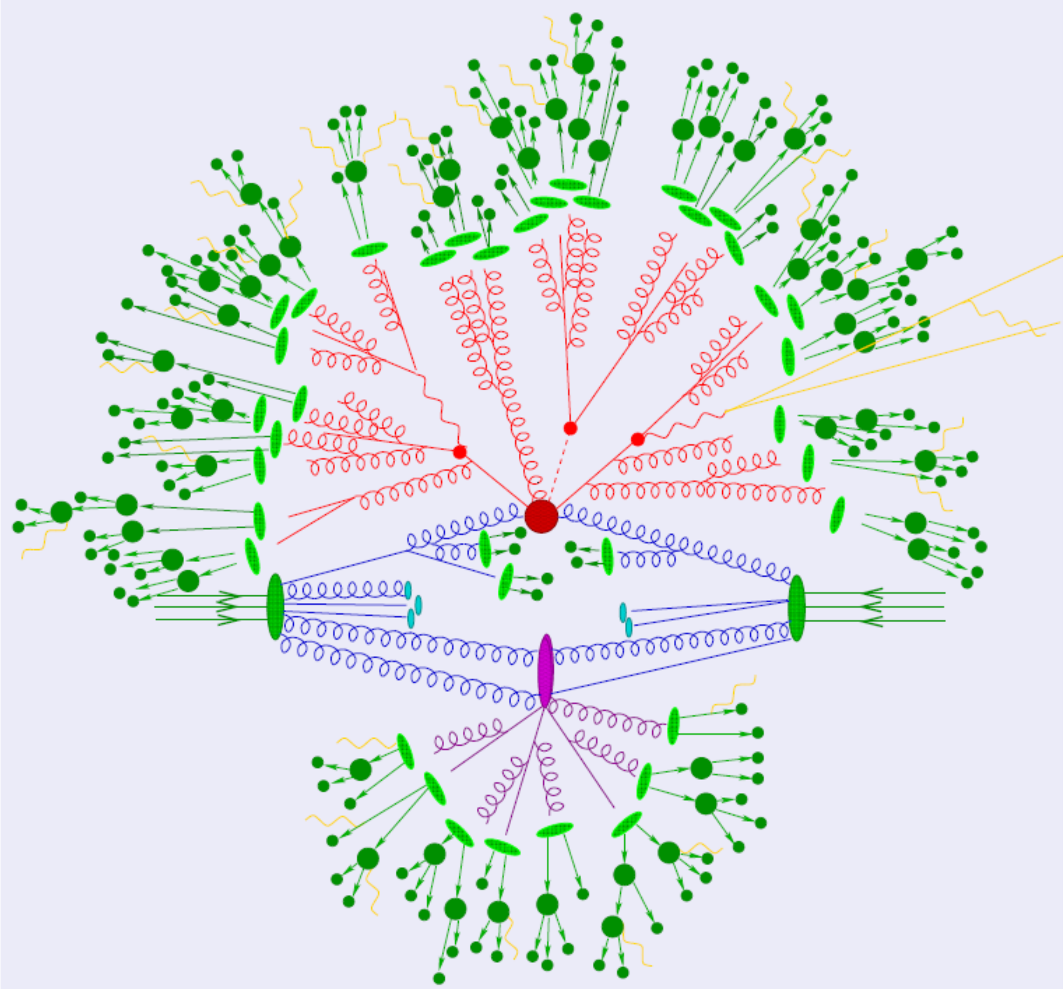
Outline

- Lessons from perturbative QED?
- Comments on Ritus-Narozhny conjecture
- Lessons from lattice QCD/QED?

Lessons from perturbative QCD, e.g. for LHC?

- Seems like a very different setup: Premium on many external particles, no external fields. Quarks and gluons massless.

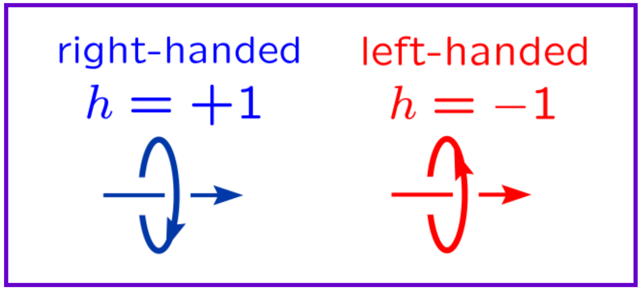


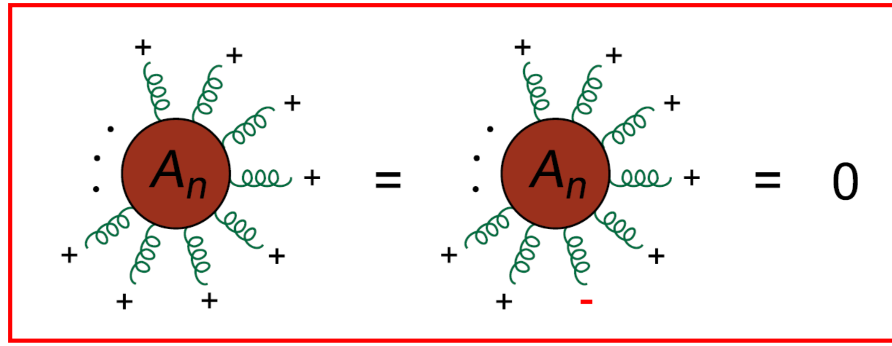
Interesting all-n helicity properties

Grisaru, Pendleton, van Nieuwenhuizen (1977)

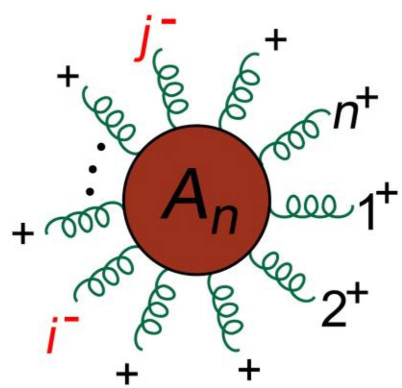
Many tree-level multi-gluon helicity amplitudes either vanish or are very short

All outgoing helicity convention





Identities derived using supersymmetry (superpartners don't contribute!).
Can also replace 2 gluons by massless quarks

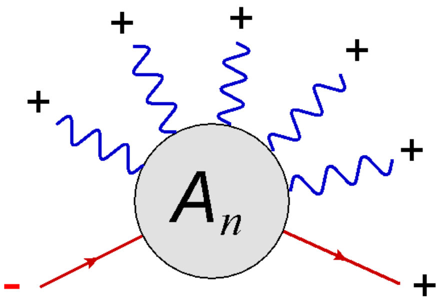


$$= \frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

Parke-Taylor formula (1986)

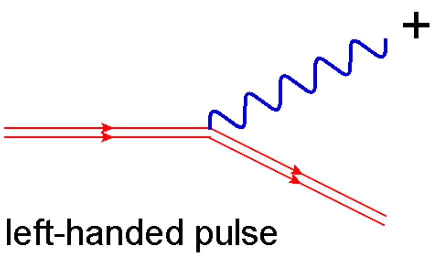
QED application

Similarly,

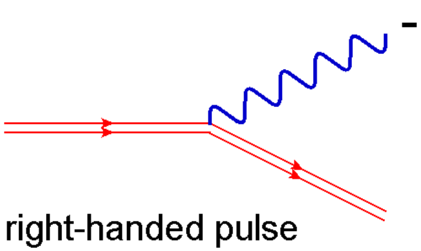


$= \ 0$ for $m_e \to 0$

- Holds for any photon and electron momenta.
- Now let all but one of the photon momenta be the same, i.e. come from same circularly polarized laser beam. Implies:



$= \ 0 \ =$



for $m_e \to 0$

Some questions

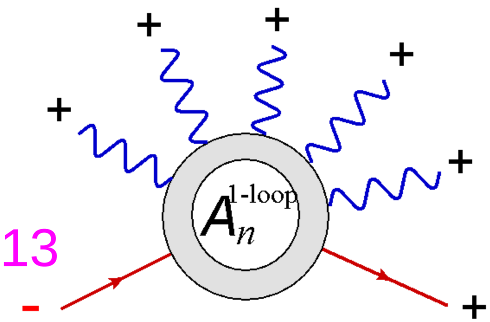
- Is this (semi)-strong field relation known already?
- Can it be tested experimentally?

(“Test of supersymmetry”?)

- With optical lasers (1eV, 1000nm) need $E_e > 500$ GeV to get $s_{e\gamma} \gg m_e^2$
- With 10nm bunch lepton collider, should be in this regime, but circular polarization??
- Focused LCLS beam (1keV, 1nm) [talk by Claudio Pellegrini] only needs $E_e > 1$ GeV. Circular polarization? How to measure outgoing γ polarization?

Quantum corrections?

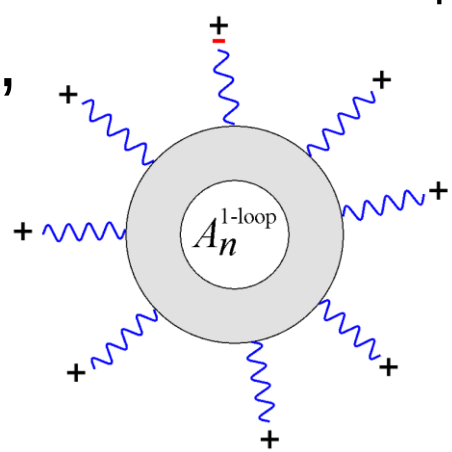
- Amplitudes that vanish at tree level due to supersymmetry generically become nonvanishing at one-loop (although they have vanishing branch cuts).
- Indeed,



$\neq$ 0

G. Mahlon, 9311213

- However,



= 0

except for $n = 4$

both for $m_e \to 0$

also Bern, Chalmers, LD, Kosower, hep-ph/9312333 and BDK, hep-ph/0505055

cf. Dinu,Heinzl, Ilderton, Marklund, Torgrimsson, 1312.6419 for b_0 >>1?

Lessons from strong-field QED for QCD?

Gluon helicity flip in a plane wave background

1903.01491

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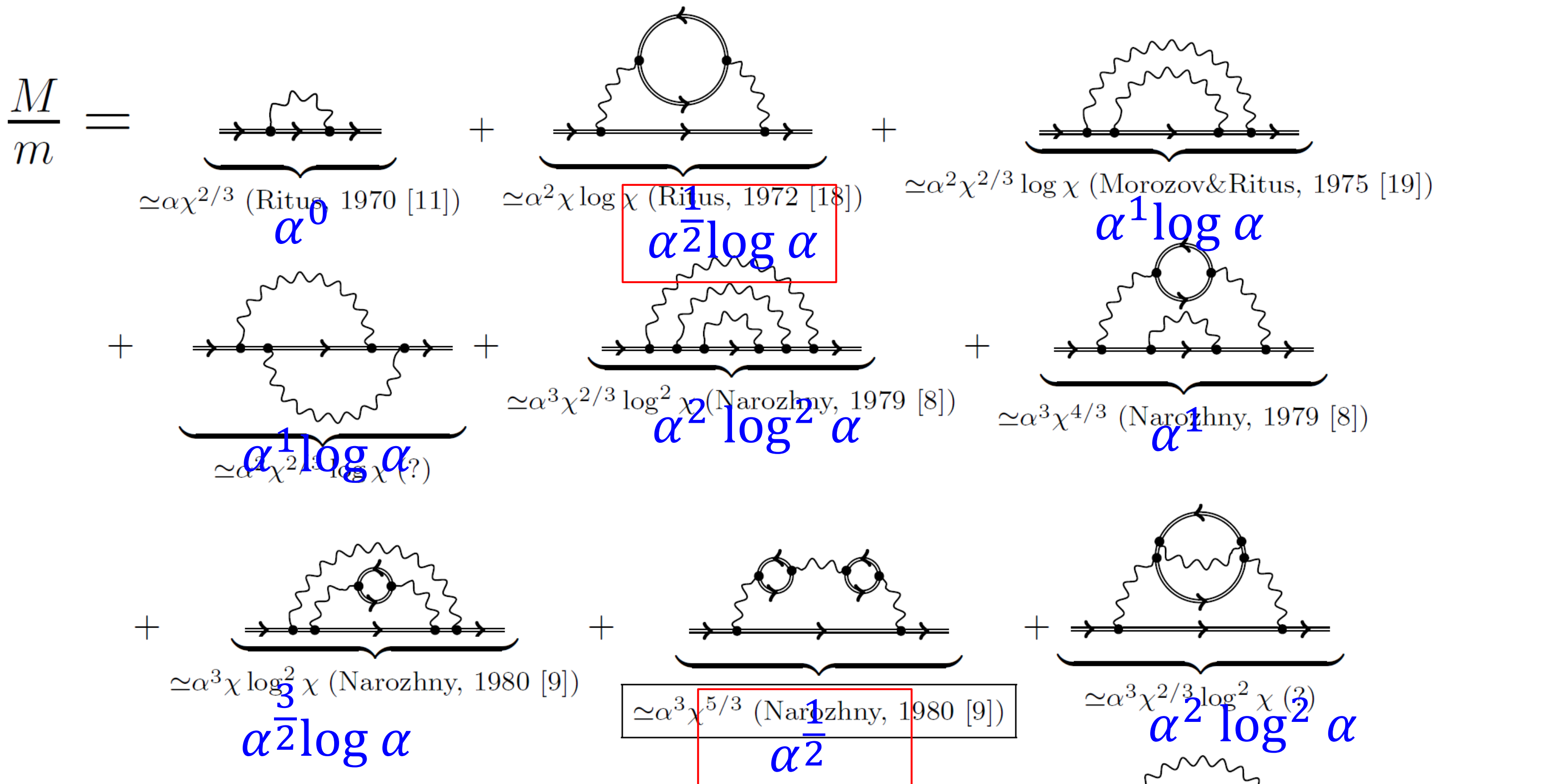
ABSTRACT: We compute the leading probability for a gluon to flip helicity state upon traversing a background plane wave gauge field in pure Yang-Mills theory and QCD, with an arbitrary number of colours and flavours. This is a one-loop calculation in perturbative gauge theory around the gluonic plane wave background, which is treated without approximation (i.e., to all orders in the coupling). We introduce a background–dressed version of the spinor helicity formalism and use it to obtain simple formulae for the flip amplitude with pure external gluon polarizations. We also give in-depth examples for gauge group SU(2), and evaluate both the high- and low-energy limits. Throughout, we compare and contrast with the calculation of photon helicity flip in strong-field QED.

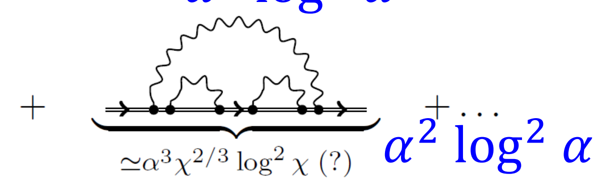
Comment on Ritus-Narozhny conjecture

Fedotov, 1608.02261, and talk yesterday

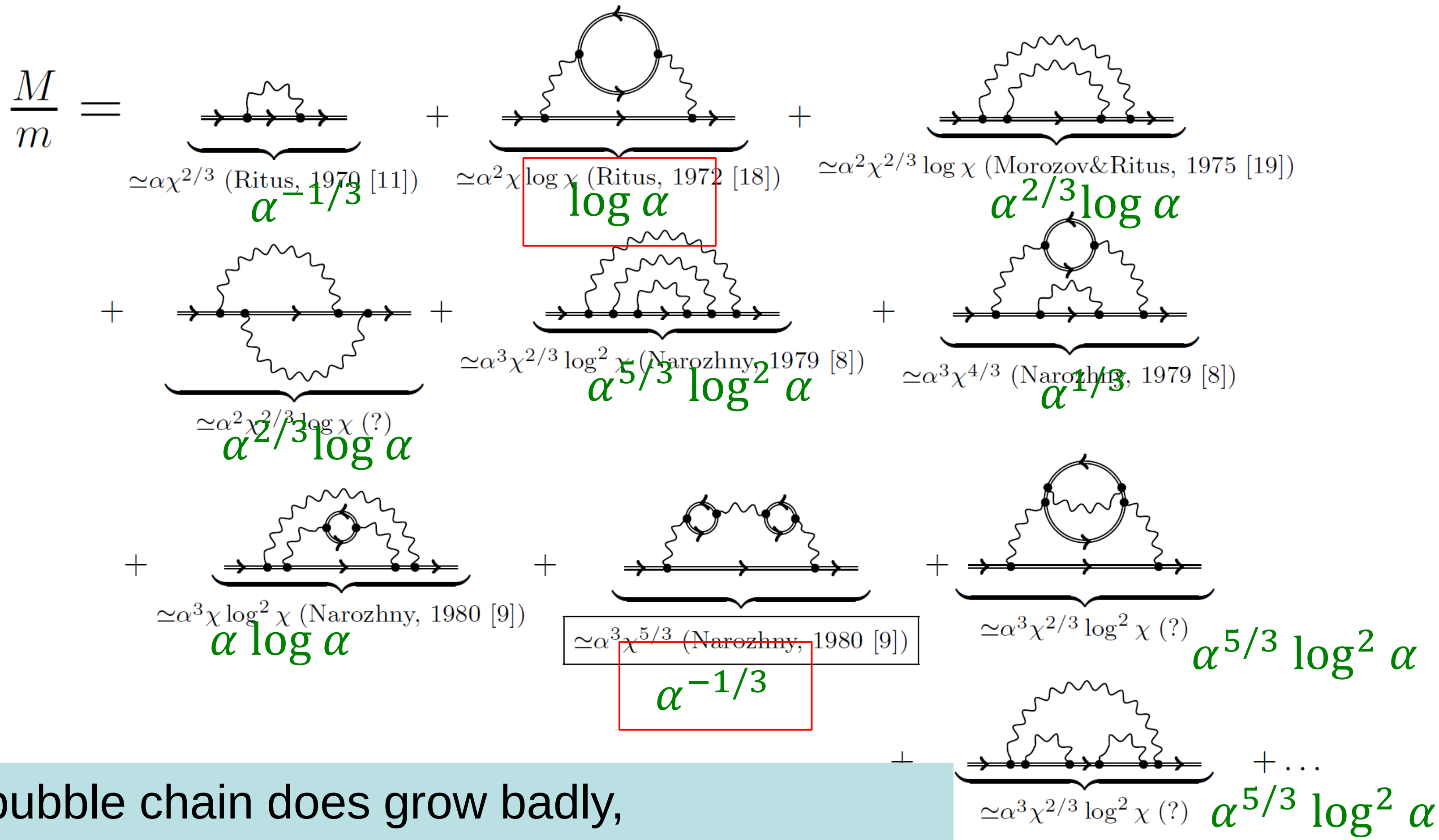
Let $\chi = \alpha^{-3/2}$



Then all graphs so far suppressed by $\alpha^{\frac{1}{2}}$
except first…(including bubble chain).



What if $\chi = \alpha^{-2}$?



Then bubble chain does grow badly, parametrically. (Consistent with behavior of resummed results shown yesterday by A. Fedotov)

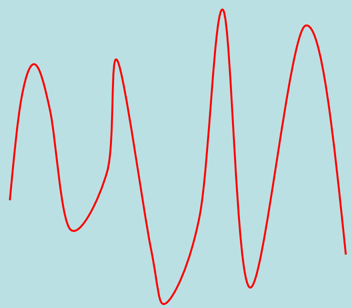
Why might [lattice] QCD/QED help?

- Lattice gauge theory is nonperturbative.
- Successfully used to compute many quantities in hadronic physics:
- hadron masses
- Matrix elements between hadrons, including transition form factors
- hadronic contributions to muon anomalous magnetic moment
- There are also at least 2 formulations of QED on the lattice (compact and noncompact).

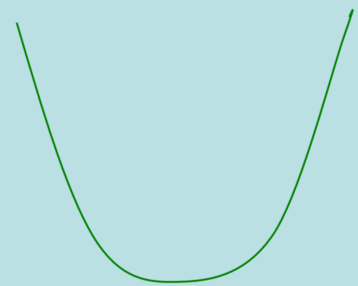
What the lattice does very well

- Simulates gauge theories nonperturbatively in Euclidean spacetime (imaginary time):

$$Z = \int [dU(x)] e^{i S_E[U(x)]} \rightarrow \int [dU(x)] e^{-S_E[U(x)]}$$



vs.



High-dimensional integrals can be done numerically by Monte Carlo integration – if the integrands are **real** and they don’t wiggle too much. “Sign problem”

Real world is Minkowski

- Sometimes this does not matter so much, for example, computing the mass of a single, low-mass hadron:
- $\langle O(0)O(t)\rangle = \sum_n \langle 0|O(0)|n\rangle\, e^{-iM_n t}\langle n|O(t)|0\rangle$

$$\rightarrow\ \sum_n \langle 0|O(0)|n\rangle\, e^{-M_n t}\langle n|O(t)|0\rangle$$

$$\sim e^{-M_1 t}\ \text{ as } t \rightarrow \infty$$

- In Euclidean time, states decay exponentially instead of oscillating. Can determine mass of lightest hadron in a given sector reliably by waiting until heavier ones have all decayed away.

Two hadrons

- In Minkowski spacetime, hadron-hadron scattering has complicated imaginary phases.
- But they affect the energy of the system in finite volume, and the volume dependence can be used to extract the Minkowski scattering amplitude. Lüscher, 1986, …
- This is even being attempted for three hadrons, e.g. Briceno, Hansen, Sharpe, 2016,…
- But trying to do any more hadrons directly seems out of reach in QCD.
- Also background electric fields will be problematic
- Can one ask relevant "NPQED" questions for only a few quanta, and only magnetic fields?

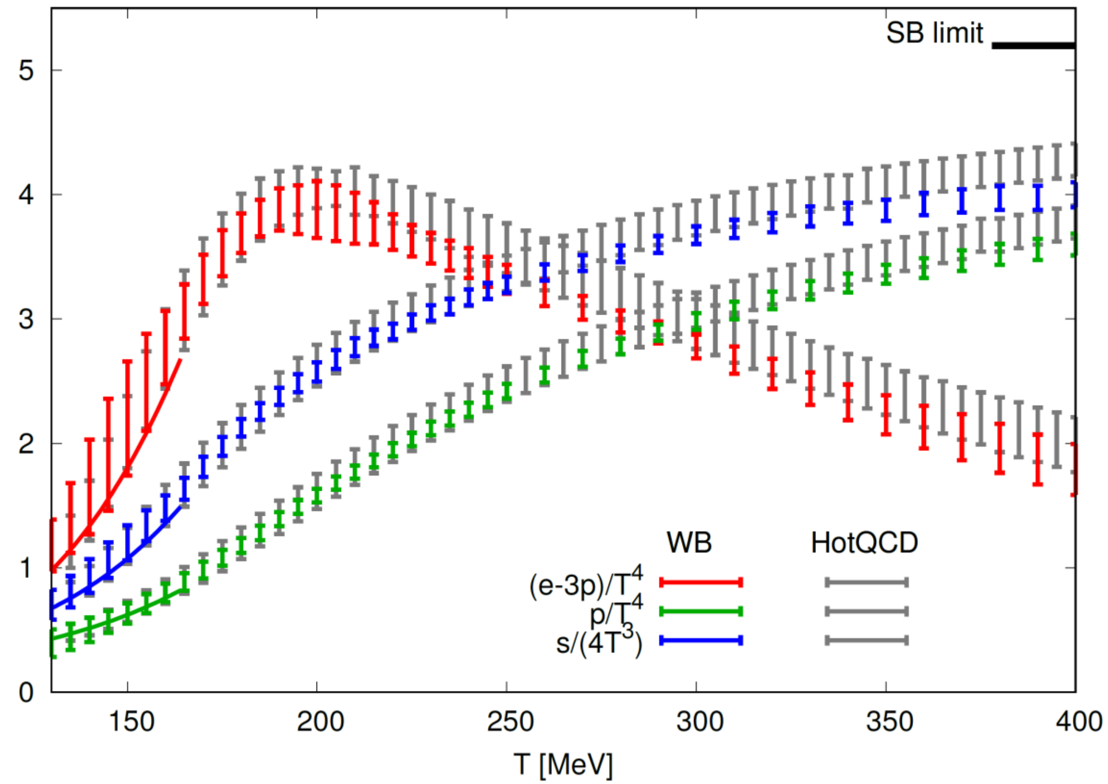
Thermal equilibrium

- Lattice QCD used for quark-gluon plasma studies, assuming thermal equilibrium and computing mainy equilibrium quantities for ~ 0 baryon number
- $e^{-\beta H} \longleftrightarrow$

Euclidean time periodic with period

$\beta = 1/kT$

Review by Ratti, 1804.07810
Lattice data for trace anomaly, entropy, pressure from HotQCD and Wuppertal Budapest (WB) collaborations



Could similar results be useful here?

- Would have to believe some kind of equilibrium was reached, and ~ 0 electron number. (Seems to be ~OK in heavy ion collisions, despite very short time scales and many initial baryons. But equilibration times are also very short.)
- What about doing it in a large background (static?) field? Background B field probably easier (done for QCD)

Even more exotic options for nonperturbative methods from HE theory?

- Integrability? Used to solve 1+1 dimensional Schwinger model.
- Relevant in 3+1 dim's for very high B limit I. Shovkovy
- Most applications of integrability are in 1+1, but planar N=4 super-Yang-Mills is one case in 3+1.
- Also AdS/CFT = holography has been used to gain qualitative information about other strongly coupled QFTs. How about NPQED?

Thanks for your time & patience!