The (anomalous) Magnetic Moment of the Muon

David Hertzog
University of Washington
January 12, 2018
To briefly setup the story for today …

\[ \Delta a_\mu \text{ (Expt – Thy)} \times 10^{-10} \]

- BNL
- Fermilab goal
- \( \sim 3.6 \sigma \)
- \( \times 2 \) Thy estimate
- \( \times 4 \)

\[ a_\mu \text{ is now measured to 540 ppb; Goal is 140 ppb} \]
But first, how I “knew” Sidney Drell …

Grad school thought: “*Whatever these things were, I wanted to know …*”

Current thought: “*I need to understand this 700+ page book since I’m making the measurement …*”
Start with Dirac equation for electron in an EM potential

\[ i(\partial_\mu - ieA_\mu(x))\gamma^\mu\psi(x) = m\psi(x) \]

- Anticipates antiparticles (which were later found)
- Predicts \( g = 2 \), as observed in atomic fine-structure experiments for the spin-1/2 electron magnetic moment (whereas an orbital picture \( \Rightarrow g = 1 \))

\[ \vec{\mu} = g \left( \frac{Qe}{2m} \right) \vec{s}, \quad e > 0 \]

- And, it allows for a so-called Pauli interaction term to accommodate possible deviations of \( g \) from 2

At first, \( g \approx 2 \) was observed. But later, the proton ...

\( g_p = 5.59 \)

and then the neutron

\( g_n = -3.8 \)

each showed large magnetic moments (\( g \neq 2 \) by a lot)

**The neutron? It’s not even charged!**

These are “Anomalous” magnetic moments owing to substructure

\[ g = 2(1 + a) \quad \text{or} \quad a = \frac{g - 2}{2} \]
Leading order in QED

\[ a_e = \frac{(g - 2)}{2} \approx \frac{1}{2 \pi} \approx \frac{1}{800} \]

\[ \Rightarrow 116,140,973.301 \times 10^{-11} \]

In 1947, small deviations from \( g = 2 \) for the “pointlike” electron were observed at about the \( \sim 0.1\% \) level.
In response to this unique Feynman challenge:

- It seems that very little physical intuition has yet been developed in this subject. In nearly every case we are reduced to computing exactly the coefficient of some specific term. We have no way to get a general idea of the result to be expected. We have no physical picture by which we can easily see that the correction is roughly $\alpha/2\pi$ in fact, we do not even know why the sign is positive (other than by computing it).

- We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a specific challenge, is there any method of computing the anomalous moment of the electron which, on first rough approximation, gives a fair approximation to the $\alpha$ term and a crude one to $\alpha^2$ ...
Higher order QED!

Presently: QED thru tenth-order terms (12,672 diagrams)

\[ a_\mu (\text{QED}, \alpha (a_\epsilon)) = 1 \ 165 \ 847 \ 188.41 \ (7)(17)(6)(28) \times 10^{-12} \]

Revised and Improved Value of the QED Tenth-Order Electron Anomalous Magnetic Moment

Tatsumi Aoyama,¹,² Toichiro Kinoshita,³,⁴ and Makiko Nio²

(Dated: December 19, 2017)
Now add in Electroweak loops
(e.g., replace $\gamma$ with a $Z$ … etc)

Well known now, but not an easy calculation

Weak 1st Order 194.820
Weak 2nd Order -41.760 $\times 10^{-11}$
And now Hadrons … which cannot be calculated using perturbation theory because of the strong coupling. This contribution can be exactly linked to experimental data:

1. Cut diagram down middle
2. Looks like $\gamma \rightarrow \pi\pi$
3. Dispersion relation connects $e^+e^- \rightarrow \pi\pi$ cross section measurement to anomaly contribution of $1^{st}$-order HVP

$$a_{\mu}^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m^2_\pi}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \text{muons})}$$
The cross sections scan a wide range in energy

Examples above is from BaBar, but many collaborations are involved ...

... and even the lattice

- KLOE (Frascati)
- SND and CMD2 (Novosibirsk)
- BES III (Beijing) ... (and very recently CLEO folks posted a contribution)
Connected and Leading Disconnected Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment with a Physical Pion Mass

Thomas Blum,1,2 Norman Christ,3 Masashi Hayakawa,4,5 Taku Izubuchi,6,2 Luchang Jin,3,7 Chulwoo Jung,6 and Christoph Lehner6

We find \( a_\mu^{\text{HLbL}} = 5.35(1.35) \times 10^{-10} \), where the error is statistical only. The finite-volume and finite lattice-spacing errors could be quite large and are the subject of ongoing research. The omitted disconnected graphs, while expected to give a correction of order 10%, also need to be computed.

In $10^{-10}$ units $\Rightarrow 5.35 \pm 1.35$, to be compared with $10.5 \pm 2.6$ usual HLbL from models.
Standard Model contributions to $a_\mu$ ... updates $\rightarrow 3.6 \sigma$

<table>
<thead>
<tr>
<th>QED $(\gamma + \ell)$</th>
<th>11 658 471.8951 ± 0.0009 ± 0.0019 ± 0.0007 ± 0.0077$_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVP(lo) Davier17</td>
<td>692.6 ± 3.33</td>
</tr>
<tr>
<td>HVP(lo) KNT2017</td>
<td>693.9 ± 2.6</td>
</tr>
<tr>
<td>HVP(ho) KNT2017</td>
<td>$-9.84 \pm 0.07$</td>
</tr>
<tr>
<td>HLbL Glasgow</td>
<td>This is a fancy guess; it will improve $\rightarrow 10.5 \pm 2.6$</td>
</tr>
<tr>
<td>EW</td>
<td>$15.4 \pm 0.1$</td>
</tr>
<tr>
<td>Total SM Davier17</td>
<td>11 659 181.7 ± 4.2</td>
</tr>
<tr>
<td>Total SM KNT17</td>
<td>11 659 182.7 ± 3.7</td>
</tr>
</tbody>
</table>

BNL E821 $\delta a_\mu(\text{Expt}) = \pm 6.3$
What could it mean if Expt ≠ Theory at > 5\(\sigma\)?

Generically, “loop effects” couple to the muon mass and moment in similar fashion, characterized a coupling, \(\propto C\)

\[
\mathcal{O}(C) \left( \frac{m_\mu}{M} \right)^2
\]

\[
C = \frac{\delta m_\mu(\text{N.P.})}{m_\mu}
\]

Following Czarnecki, Marciano, and Stockinger

Could it mean if Expt ≠ Theory at > 5\(\sigma\)?

If Expt ≠ Theory at > 5\(\sigma\), it could mean:

- \(\mathcal{O}(1)\):
  - Radiative muon mass generation...
    - [Czarnecki, Marciano '01]
  - [Crivellin, Girrbach, Nierste '11][Dobrescu, Fox '10]

- \(\mathcal{O}(\frac{\alpha}{4\pi})\times\text{Factor}\):
  - Supersymmetry \([\tan\beta]\)

- \(\mathcal{O}(\frac{\alpha}{4\pi})\):
  - Vectorlike fermions...

- \(\frac{\alpha}{4\pi}\) < 
  - SM: Z, W. New physics: Z', W'...
  - 2-Higgs doublet model, dark photon.

Following Czarnecki, Marciano, and Stockinger
And the situation now?

Some things “seen” just wash away ... And some things are under Tension

LHC limits growing, but SUSY, if exists, is hiding well

And some things don’t’ seem to be so ...

g-2: An uncomfortably lonely search for a Crack in the SM
The Measurement

Muon g-2 Collaboration
(190 collaborators, 34 institutes, 7 countries)

We hail from: Particle-, Nuclear-, Atomic-, Optical-, Accelerator-, and Theory-Physics Communities

But, we are all measuring g-2
The Fundamental Principle

\[
\omega_a = \frac{q}{m} a_{\mu} B
\]

Determine difference between spin precession and cyclotron motion for a muon moving in a magnetic field:

The expression including \( E \)-field focusing and possible \( \mu \)EDM

\[
\vec{\omega}_{net} = -\frac{q}{m} \left[ a_{\mu} \vec{B} - \left( a_{\mu} - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2} \left( \vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]
\]

Get \( a_{\mu} \)

Measure these

Magic \( \gamma \)

EDM

\[
\vec{\omega}_{net} = \vec{\omega}_a + \vec{\omega}_{EDM}
\]
A formal way to write this looks like this:

\[ \ln E_{821} \equiv \mathcal{R}_\mu(E_{821}) = 0.0037072064(20) \text{ [0.54 ppm]} \]

- \( a_\mu(\text{Expt}) = \frac{g_e}{2} \frac{\omega_a}{\tilde{\omega}_p} \)
- \( m_\mu \frac{\mu_p}{\mu_e} \)

-2.002 319 304 361 53(53) [0.26 ppt] Electron g-2 + QED

-.001519270384(12) [8 ppb]
206.768 2843(52) [25 ppb]

We will measure these two frequencies and report the Ratio
Requirements for a better measurement

1. Store More Muons
   • 21 x BNL in statistics … (100 ppb)

2. Prepare A More Uniform Magnetic Field
   • Goal → 3 x better and more carefully measured (70 ppb)

3. Improve the Precession Frequency Measurement
   • All new instrumentation with high-fidelity recording of muon decays by many systems (70 ppb)
Creating the Muon Beam for $g-2$

- 8 GeV $p$ batch into Recycler
- Split into 4 bunches
- Extract 1 by 1 to strike target
- Long FODO channel to collect $\pi \rightarrow \mu \nu$
- $p/\pi/\mu$ beam enters DR; protons kicked out; $\pi$ decay away
- $\mu$ enter storage ring

Intensity profile is 120 ns wide with “W” shape
To measuring the Field, $\omega_p$, we start with the BNL magnet but improve its field uniformity

- Yoke Iron
  Aligned to sub-mil precision

- Superconducting coils
- And cryostat

- Magnet shimming kit
  - NMR probes
  - Probe Multiplexer
  - Pulser-Mixer
24 Calorimeter stations located all around the ring

NMR probes and electronics located all around the ring
Built-in Shimming Tools provide many knobs

- **B Field** 1.45T
- **12 Yokes**: C shaped flux returns
- **72 Poles**: shape field
- **864 Wedges**: angle - quadrupole (QP))
- **24 Iron Top Hats**: change effective mu
- **Edge Shims**: QP, sextapole (SP)
- **8000 Surface iron foils**: change effective mu locally
- **Surface coils**: will add average field moments (360 deg)

**g-2 Magnet in Cross Section**
The Field is measured using a proxy: pulsed NMR of protons

Free Induction Decay (FID) Waveforms

Extracted frequency precision is \(~10\) ppb per FID

A 25-element pNMR *Trolley* maps the field during shimming.

You see the probes here as the trolley goes around the ring in azimuth
Start by (painfully) aligning only pole surfaces:

- **Red** is Initial dipole field starting point at Fermilab
- **Blue** is typical BNL final field *after* shimming
Then adjust only Top Hats and Wedges
The big and final breakthrough idea: Iron Laminations:

- Cover each pole with a patchwork of foils in 41 azimuthal and 3 radial sections

- Determine optimal foil mass values by iterative procedure that minimizes field inhomogeneity around a target value – Dave Kawall U Mass
After adding the 8000 iron foils, the precision field emerges.

The Goal is Achieved!

This is only the start of a full story ... it now must be measured precisely and mapped.
Measuring the precession frequency, $\omega_a$ by sorting $e^+$ decay Times and Energies.

Systematic error total $\omega_a$ budget: [210 ppb $\rightarrow$ 70 ppb]
Measuring the precession frequency, $\omega_a$, by sorting $e^+$ decay Times and Energies.

Systematic error total $\omega_a$ budget: [ 210 ppb $\rightarrow$ 70 ppb ]
What makes the “wiggle”? 

Threshold Energy Cut 

Energy in Calorimeters 

Fraction e^+ above Threshold 

Phase of Muon Spin
24 Calorimeter with 54 PbF$_2$ Cherenkov crystals and very fast SiPMs

Tested at SLAC Test Beam
Thanks to Carsten Hast!
GAIN stability established to \( \sim \) few \( \times 10^{-4} \)

State-of-the-art Laser-based calibration system also allows for pseudo data runs for DAQ

Inside the laser hut

(in Test Beam)

\[ \frac{10^{-4}}{h} \text{ demonstrated} \]
The experiment is completing commissioning phase soon

We are in the rare process of “christening a battleship”
First evidence of stored muon precession
24 Calorimeters Data and Simulation in detail
(and, before any fancy calibration)
Sample ** from recent days … with a statistical uncertainty, $\frac{\delta a_\mu}{a_\mu}$ of $\sim 5$ ppm

**Warning!! this is not yet “physics” data. Many calibrations are not completed and magnetic field is not mapped during this period.**
Sample ** from recent days … with a statistical uncertainty, $\delta a_\mu/a_\mu$ of ~5 ppm

**Warning!! this is not yet “physics” data. Many calibrations are not completed and magnetic field is not mapped during this period.
Energy vs Time shows muon precession and muon coherent betatron oscillations (from losses).

Muon coherent betatron oscillations, since by muons that get kicked out by collimators and punch through calorimeters (MIPs).
In-vacuum Straw Tracker determines Muon Distribution needed for the “~” in $\omega_p$ formula.
The off-momentum muon spins are slightly affected by the radial $E$ field

\[ \vec{\omega} = -\frac{q}{m} \left[ a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} + (-a_\mu + \frac{1}{\gamma^2 - 1}) \frac{\vec{\beta} \times \vec{E}}{c} \right] \]

\[ \vec{B} = B \hat{j} \]

$\beta \times E$ and "$\gamma$" terms signs both flip depending on momentum

$\rightarrow$ No cancellation

$\rightarrow$ All off-momentum muons reduce effective $a_\mu$

$\sim 0.5$ ppm effect, net
The “pitch” correction owing to vertical betatron oscillations

\[
\ddot{\omega} = -\frac{q}{m} \left[ a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} + (-a_\mu + \frac{1}{\gamma^2 - 1}) \frac{\vec{\beta} \times \vec{E}}{c} \right]
\]

(great exaggeration)

Pitch effect reduces the magnitude of \( \omega_a \)

\[
\left\langle (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right\rangle = \left\langle \beta_y B (\beta_x \hat{i} + \beta_y \hat{j} + \beta_z \hat{k}) \right\rangle = \left\langle \beta_y^2 \right\rangle B \hat{j}
\]

\[C_P = \frac{\langle \psi^2 \rangle}{2} = n \frac{\langle y^2 \rangle}{(2 R_0^2)}\]

\(~0.25 \text{ ppm effect, net~}~\)
Outlook

• The search for New Physics can profit from the “Indirect” approach … as long as the measurement is very precise and the Standard Model expectations are clear

• We are presently in a bit of an LHC lull … we anticipated more, but haven’t seen it yet
  – The “TeV Scale” is so far not bearing unexpected fruit

• A few very sensitive experiments are pushing the envelope, but we don’t yet know what will tip over the vase
  – EDMs ?
  – cLFV searches?
  – $0\nu\beta\beta$ programs ?
  – Beta decays? PVES ?
  – MUON g-2 !!!

STAY TUNED!
E989 Scientific collaboration

Domestic Universities
- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- Northern Illinois
- Regis
- UT Austin
- Virginia
- Washington

• National Labs
- Argonne
- Brookhaven
- Fermilab

Italy
- Frascati
- Molise
- Naples
- Pisa
- Roma 2
- Trieste
- Udine

England
- Lancaster
- Liverpool
- University College London

Korea
- CAPP/IBS
- KAIST

China
- Shanghai

Germany
- Dresden

Russia
- JINR/Dubna
- Novosibirsk

7 countries
35 institutions
~192 authors
Optimizing Statistical Error

\[ \delta \omega_a = \frac{1}{\gamma \tau_\mu} \sqrt{\frac{2}{N \Lambda^2}} . \]
Muon Primer

- Mass $\sim 207 \, m_e$ (50 ppb)
  - $(m_\mu/m_e)^2 \approx 43,000$ times more sensitive to “new physics” through quantum loops compared to electrons (taus would be better!)

- Lifetime $\sim 2.2 \, \mu$s (1 ppm)
  - High-intensity beams; can stop and study; can possibly collide

- Primary production: $\pi^+ \rightarrow \mu^+ \nu_\mu$ (99.98%)
  - Polarized naturally:

- Primary decay $\mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu$ (~99%)
  - Purely weak; distribution in $\theta$ and $E$ reveals weak parameters

- Lepton number is conserved
  - (BRs < $10^{-12}$)
Two “blinded” frequency measurements are made. The ratio gives $a_\mu \equiv (g-2)/2$

(1) Precession frequency
   (1) Calorimeters

(2) Muon distribution
   (2) Trackers & Models

(3) Magnetic field
   (3) proton pNMR

How do we get each of these?
The attractive idea: SUSY

Difficulty to measure at the LHC

Recall, the deviation between Experiment and Theory is \( \sim 280 \times 10^{-11} \), so the above calculation is interesting if you put in \( M_{\text{SUSY}} \), and \( \tan\beta \)

\[ a_\mu^{\text{SUSY}} \approx 130 \times 10^{-11} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan\beta \ \text{sign}(\mu) \]

tan\(\beta\)? Ratio of the two vacuum values of the 2 neutral Higgses, typically estimated in range from 3 to 55
Equipment to provide muon injection and storage

The steps:

1. Steer beam through SR magnet backleg corridor and through **Inflector** into ring
2. Provide well-timed angular **kick** to direct muons into storage region
3. Asymmetrically power **quads** to gently scrape edge muons on **collimators**
4. Relax quads to symmetric configuration for stable vertical containment
The injection hardware

Inflector field cancels the main g-2 magnet

View of Inflector

Muons ent

Kicker Plates

Kicker Pulse

Triaxial Blumlein Kicker Tubes Mounted and Connected in MC1